# Exploring Students' Understanding on 'Inequalities'

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#### Abstract

The topic on Inequalities is one of the main topics in Year 9's secondary mathematics scheme of work in Brunei Darussalam which consists of three sub-topics: Meaning and Symbols, Solve Linear Inequality and Graphical Representation of Inequalities. The primary focus for this study is to investigate which area or items that the students find easy or difficult. Twenty test items were prepared and the questions were adapted from textbooks and past year questions. The two items (10%) tested the students' understanding on the meaning and symbols in inequalities, fifteen items (75%) on solving linear inequality and three items (15%) on graphical representation. A total of 51 students from two different classes were given the test on the same day. The two classes were from the General Science Programme and the General Programme. Using the Discriminative Indea and Percentage of Difficulty analysis shows that students are confused with the value of negative integers, made careless mistakes and had poor basics in solving algebraic equations. In order to improve the students' performance on the test, we recommend the teacher should first assess the students' prior knowledge especially with integers and algebra before proceeding with the lesson on inequalities.

Keywords: Test item analysis, Secondary mathematics, Inequalities, Brunei Darussalam

## 1. Introduction

The topic on 'Inequalities' is one of the main topics in the Year 9 mathematics scheme of work in the secondary schools in Brunei Darussalam. In this study, the Year 9 students were given a test on inequalities in order to investigate which area or items that the students find easy or difficult. In addition to items difficulty, suggestions on ways to improve the students' performance on this topic will also be discussed. According to the subject teacher's scheme of work, the topic on inequalities consists of three sub-topics: Meaning and Symbols, Solve Linear Inequality, and Graphical Representation of Inequalities. Table 1 below shows the learning objectives for each sub-topic extracted from the scheme of work provided by the school.

Table 1: Learning objectives of the content coverage

Content coverage	Learning Objectives
Meaning and symbols	<ul> <li>Define the symbols used in inequalities: '&lt;' means less than, '&gt;' means greater than, ' ≤ ' means less than or equal to and '≥' means greater than or equal to.</li> <li>Compare the size of two numbers using the symbols '&lt;'and'&gt;'.</li> </ul>
Solve Linear Inequality	<ul> <li>List the values of a linear inequality such as x ≥ 1, x ≤ 2, -2 &lt; x ≤ 3.</li> <li>Represent the linear inequality on a number line and vice versa. (Emphasize that for &lt; or &gt; use a circle or dotted vertical line to mark the end point whereas for ≤ or ≥ use a dot or solid vertical line to mark the end point).</li> <li>Solve linear inequalities in one variable.</li> <li>Solve simultaneous linear inequalities in one variable.</li> <li>Determine the possible solutions or solution set of a given inequality under various conditions.</li> <li>Find the least and greatest sum, difference, product and quotient of two variables given in two separate inequalities (include their squares).</li> </ul>

Review sketching of straight lines and writing equations for lines in a given diagram.
 Remind students about the convention in using solid and dotted lines and indicate by a sketch, the region defined by an inequality (usually by shading the unwanted region).
 Write the inequalities, which define a region (usually un-shaded) where the equations of the boundary lines are given or not given.
 Explain how to obtain the region defined by a system of linear inequalities.
 Determine the maximum or minimum of ax + by for a defined region by evaluating the expression at the vertices of the polygon formed by the region.

#### 2. Review of the Literature

Frempong (2012) defined inequalities as a mathematical statement that consisted of two expressions that are not equal. Davies and Peck (1855) stated that in algebra, the expressions of two unequal quantities are connected by a symbol. The symbol here refers to <, <, >, >, <, >, >, = and = and these symbols represent a relationship between two parts of the inequalities: the first member is left of the sign of inequality and the second member is right side of the inequalities.

The sense of inequality refers to whether the inequality symbol is the greater than symbol > or the less than symbol < (Frempong, 2012). The solution set derived through solving linear inequalities "is the set of real numbers each of which when substituted for the variable makes the inequality true" (Frempong, 2012, p. 121). In other words, finding all value of the variable for which the inequality is true or that satisfy the inequality (Larson, Hostetler & Edwards, 2008). Frempong also pointed out that solving linear inequalities are "similar to the techniques for solving linear equations, except that when an inequality is divided or multiplied by a negative number, the sense of the inequality must be reversed" (p. 121).

# 2.1 Equivalence involved in solving linear inequalities

Li (2007) conducted a research on 'An Investigation of Secondary School Algebra Teachers' Mathematical knowledge' identified three types of equivalence involved in equations and equation solving. As solving linear inequalities is said to be similar (Frempong, 2012) to solving linear equations, it is safe to say that these equivalence applies to Li's findings as well. And these are given below.

- 1. Equivalence involved within equations that link variable expressions
- 2. Equivalence among algebraic expressions that are connected by transformations which preserve values
- 3. Equivalence between the original equation and those derived in the solving process.

As mentioned by Li, many students are not aware of these distinctions (Greeno, 1982). Students fail to understand that solutions that they obtained have meanings or relevance to the equation or to understand the underlying properties or the meaning of the equality (Kieran & Sfard, 1999). Furthermore, Lim (2006) stated that as a result of students being taught to treat inequalities as equations, they might not be required to grapple with the meaning of the solution set.

The nature and levels of development of a precept are dependent on the cognitive growth and experiences of the child (Anghileri, 2000). Anghileri believed that the more varied the patterns that children can readily associate with symbols; the better will be their preparation for incorporating these ideas into problem-solving strategies. These days, Mathematics in general, is seen as set of facts and algorithmic procedures and students find it difficult to grasp the meaning of new concepts in the classroom due to the lack of relevant experience from everyday life (Lave, Smith & Butler, 1989). Furthermore, Tsamir and Bazzini (2004) found that students' intiuitive beliefs when solving inequalities interferes with their mathematical decisions hence their performance. They further stated that the students mainly believed intuitively that "inequalities must result in inequalities and that solving inequalities and equations are the same process" (Tsamir & Bazzini, 2004, p. 809). Recommendations include teachers addressing this belief and encouraging them to search for ways to overcome this issue.

#### 3. Methodology

The main objectives of this study are to investigate students' understanding in the topic of inequalities and to explore the nature of difficulties encountered in this topic area. There were 20 test items prepared and the questions were adapted from textbooks and past year questions (refer to Appendix A for the test questions). The distribution of the 20 items for each sub-topic is in the Test Blueprint presented in Table 2.

Table 2: The distribution in the Test Blueprint

Sub-Topic	Test Item Number	Ideal Percentage of Items
Meaning and symbols	1, 2	10%
Solve Linear Inequality	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13(a), 13(b), 14(a), 14(b), 14(c)	75%
Graphical Representation of Inequalities	15(a), 15(b), 16	15%

The two items (10%) tested the students' understanding on the meaning and symbols in inequalities. The two main symbols used in inequalities, the less than sign (<) and the more than sign (>) were given to test the students in comparing between two integers which are positive and negative. The students were also expected to represent the inequalities on a number line. The sub-topic under solving linear inequality had the highest ideal percentage (75%) among the three sub-topics. This is because the main learning objectives are focused in the sub-topic of solving linear inequality. The items given in this sub-topic were about solving different types of inequalities and finding greatest or smallest possible values from given inequalities. The final three items (15%) tested on graphical representation and these three items were extracted from the General Cambridge of Education Ordinary Level or the GCE O Level past year questions. These questions were extracted from Paper 1 to see if the students were able to answer the questions designed for the O Level exams.

Before conducting this study, permissions were sought from the relevant authorities at the school and ministry levels. Only when access was granted, a test was given to a total of 51 Year 9 students (14-15 year olds) from two different classes, and the test was disseminated on the same day. The two classes were from the General Science Programme and the General Programme. These two classes are different where students were streamed based on their Student Progress Assessment (SPA) results in their Year 8. In the streaming process, the particular school looked at the students' grades for each subject including mathematics. For the students in the General Science Programme, they scored 50% and above for mathematics, while those in the General Programme scored 31% to 49% for mathematics in the Year 8 SPA results. The same teacher taught these students and they completed the lessons on inequalities a week before the test was administered. The students were given 40 minutes to complete the test and they were not allowed to use calculators. The students' answers were marked as either correct or wrong for each of the test items.

## 4. Results

The 'Students Achievement' and 'Discrimination Index' and 'Percentage of Difficulty' obtained from the raw data of the test are considered in reporting the results. Table 3 represents the students' achievement for all the 51 participants. The total marks and percentage were arranged from the highest and lowest and the students were divided into three groups: the Higher Group, the Average Group and the Lower Group. The groups were divided as one-third for each group. The sample used for this analysis was taken from the 17 answers from the higher group and 17 answers from the lower group. Note that all the students' names have been kept anonymous for ethical purposes and only presented as numbers. The highest mark is 11 (50%) and the lowest is 1 (5%). The average mark is 22%, which is considered low and unexpected. The range score is 45%, the mode is 3 (14%) and the median mark is 5 (23%).

**Table 3:** The students' achievement categorised according to group level

Groups	Number	Students	Class	Total correct answers out of 20 questions	Percentage marks score
	1	Student 12	Science	11	50%
	2	Student 1	Science	10	45%
	3	Student 20	Science	10	45%
	4	Student 6	Science	9	41%
	5	Student 13	Science	9	41%
	6	Student 33	General	9	41%
Higher Group	7	Student 14	Science	8	36%
	8	Student 15	Science	8	36%
	9	Student 19	Science	8	36%
	10	Student 16	Science	7	32%
	11	Student 17	Science	7	32%
	12	Student 24	General	7	32%
	13	Student 42	General	7	32%

	14	Student 4	Science	6	27%
	15	Student 9	Science	6	27%
	16	Student 10		6	27%
	17	Student 11	Science	6	27%
	18	Student 21	Science	6	27%
	19	Student 25		6	27%
	20	Student 39		6	27%
	21	Student 48		6	27%
	22	Student 5	Science	5	23%
	23	Student 7	Science	5	23%
	24	Student 8	Science	5	23%
	25	Student 23		5	23%
Average Group	26	Student 26		5	23%
	27	Student 51		5	23%
	28	Student 30		4	18%
	29	Student 37		4	18%
	30	Student 44		4	18%
	31	Student 3	Science	3	14%
	32	Student 18		3	14%
	33	Student 28	General	3	14%
	34	Student 29	General	3	14%
	35	Student 31	General	3	14%
	36	Student 32	General	3	14%
	37	Student 36	General	3	14%
	38	Student 41	General	3	14%
	39	Student 49	General	3	14%
	40	Student 2	Science	2	9%
	41	Student 38	General	2	9%
	42	Student 43	General	2	9%
Lower Group	43	Student 45	General	2	9%
	44	Student 46	General	2	9%
	45	Student 50	General	2	9%
	46	Student 22	Science	1	5%
	47	Student 27	General	1	5%
	48	Student 34	General	1	5%
	49	Student 35	General	1	5%
	50	Student 40	General	1	5%
	51	Student 47	General	1	5%

Table 4 below shows the 'Discriminative Index and Percentage of Difficulty' analysis. Discriminative index analysis refers to the potential of an item to discriminate two groups of students (higher and lower group) that represents one-third of the total students that obtained the highest and the lower scores. The difficulty percentage analysis shows the percentage of students who answered the items correctly (Hamzah & Abdullah, 2011). Discrimination index of an item is the difference of total correct of the higher group (Htc) and the total correct of the lower group (Ltc) divided by total number in higher (H)  $\frac{\sum (H+L) - \sum Htc + Ltc)}{\sum (100\%)} \times 100\%$ 

or lower group (L), which is 17. Meanwhile, the calculation for the difficulty percentage is  $\overline{\Sigma^{(H+L)}}$ 

Table 4: The discrimination index and percentage of difficulty

Item	Total correct of the higher group	Total correct of the lower	Total correct	Difference	Discrimination	Difficulty Percentage
Number	(Htc)	group (Ltc)	(Htc + Ltc)	(Htc - Ltc)	index	(%)
1	17	16	33	1	0.06	2.94
2	14	3	17	11	0.65	50.00
3	17	2	19	15	0.88	44.12
4	14	1	15	13	0.76	55.88
5	10	2	12	8	0.47	64.71
6	14	5	19	9	0.53	44.12
7	5	0	5	5	0.29	85.29
8	9	2	11	7	0.41	67.65
9	4	0	4	4	0.24	88 24

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10	13	2	15	11	0.65	55.88
11	2	0	2	2	0.12	94.12
12	7	0	7	7	0.41	79.41
13(a)	1	0	1	1	0.06	97.06
13(b)	1	0	1	1	0.06	97.06
14(a)	4	0	4	4	0.24	88.24
14(b)	1	0	1	1	0.06	97.06
14(c)	0	0	0	0	0.00	100.00
15(a)	1	0	1	1	0.06	97.06
15(b)	0	0	0	0	0.00	100.00
16	0	0	0	0	0.00	100.00

Test items 2, 3, 4, 5, 6, 8, 10 and 12 are considered as excellent items based on the interpretation of the discrimination index of test item mentioned in Hamzah and Abdullah (2011). Items 7, 9 and 14(a) are average items and item 11 is considered as unsatisfactory and needed improvement. While items 1, 13(a), 13(b) 14(b), 14(c), 15(a), 15(b) and 16 are considered poor items and should be omitted or changed. Due to time constraint, the test items were not piloted and no changes were made to the items. However, the main focus in here is the difficulty percentage. Item 1 is considered too easy while items 3 and 6 are considered as easy questions. And items 2, 4, 5, 8 and 10 are good questions. Item 12 is a difficult question while items 7, 9, 11, 13(a), 13(b), 14(a), 14(b), 14(c), 15(a), 15(b) and 16 are considered as the too difficult questions. These items are categorised based on Kolstad et al.'s (1984) guidelines for evaluating an item from the aspect of discrimination index.

The test was designed whereby the first few questions are the easiest ones and the level of difficulty increases towards the last few questions. Based on the difficulty percentage, it is shown that the distribution of the questions from too easy to too difficult was as expected. It is important to put the easy questions first to motivate the students to answer the other questions that follow (Hamzah & Abdullah, 2011). Items 1, 2, 3 and 4 are items tested for the content coverage on meaning and symbols and students were expected to be able to answer these 4 items correctly. However, only 49% of students were able to answer item 2 correctly when 98% of the students were able to answer item 1 which is a similar question as item 2.

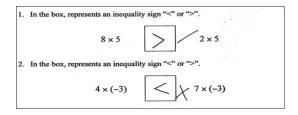


Figure 1: Student's answer to items 1 and 2

As displayed in Figure 1 above, students are confused with the value of negative integers. Most students answered negative 12 is less than negative 21 (-12 < -21). The same goes with item 4 shown in Figure 2, where most students answered by drawing the arrow to the right.

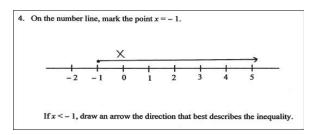


Figure 2: Student's answer to item 4

From our observations, these mistakes are common. Students knew that the value decreases as we move to the right of

the number line but they do not think before they answer. Hence, these mistakes can be considered as carelessness. Items 7 and 9 are considered as too difficult questions under the content coverage of solving linear inequalities. Majority of the students did not attempt to answer item 7, and a few of those who attempted this question (refer to Figure 3) made the same mistake of not switching the inequality sign after dividing the right-hand side with negative 2 (as was taught by the teacher).

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7. Solve the inequality -\frac{2}{3}x \le 4.
-\frac{2}{3}x \times \frac{4}{1}
2x \le 12
x \le \frac{12}{-2}
x \le \frac{12}{-2}
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Figure 3: Student's answer to item 7

For item 9, the teacher taught the students to split the inequality into two sections and to solve each inequality separately. Those who made mistakes were mostly because of the change of sign when dividing the other side with a negative integer and careless mistakes when dividing positive integer with a negative integer to form a positive value. In Figure 4 we present how one student solved item 9. The few reasons why students failed to answer correctly these two items were careless mistakes and poor basic in solving algebraic equations.

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9. Solve the inequality -5 < 2x + 3 < 1.
-5 < 2x + 3
2x + 3 < 1
2x < 5 + 3
2x < 1 - 3
2x < -2
2x < -2
2x > -4
2x < -2
2x < -2
2x < -2
2x < -2
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Figure 4: Student's answer to item 9

Items 13(a), 13(b), 14(a), 14(b), 14(c), 15(a), 15(b) and 16 are considered as too difficult items. This can be seen when most students did not attempt to answer the questions. Items 13(a), 13(b), 14(a), 14(b) and 14(c) are similar questions in determining the possible solutions or solution set of a given inequality under various conditions and to find the least and greatest sum, difference, product and quotient of two variables given in two separate inequalities. A few students who attempted these questions managed to do the first part (13(a) and 14(a)) but not the second and the relevant subsequent parts (13(b), 14(b) and 14(c)). The main reason for this is that the students did not list the integers within the set of the inequalities. They just look at the numbers in the inequality and use them to solve the largest or smallest values (refer to Figure 5 below).

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14. Given that -5 \le x \le 3 and -3 \le y \le 5, find
13. Given that -3 \le x \le 2 and -7 \le y \le 3, find
                                                  c a. the smallest value of x+y and
\bar{k} a. the largest value of x-y and
                                                            -3+3=0
        2-3=-1
     -3-3 -- 6
                                                            -3-5 =-8
                                                             5-5-0
        2 - (-7) = 9
                                                  F b. the largest value of x^2 + y^2
\bar{\mathbf{E}} b. the smallest value of x^2
  And 3-3-2-4
                                                             Ans 1. 32 + 52
                                                                  . 9 + 25
         Ans-0-3 X
                                                                  = 34
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Figure 5: Student's answers to item 13(a), 13(b), 14(a) and 14(b)

If the students listed out the values in the inequalities, they could see that x = 0 will give the smallest value for  $x^2$ . While for item 14(b) the student did not think that the square of a negative integer is a positive integer. This student just looked at 3 and 5 without considering the negative integer of x = -5.

The difficulty percentage of items 14(c), 15(b) and 16 is 100%. This meant that no students were able to answer these questions correctly. For item 15(a) only two students were able to answer the question correctly where they were able to draw the line of  $y = \frac{1}{2}x - 2$ , however they were unable to shade the region defined by the three inequalities especially for  $x \ge 0$ . This shows that the students are weak with their basic linear graphs and were unable to determine which side to shade. Hence they did not attempt to answer these questions.

Referring to Table 3 again, 14 out of 17 students were from the General Science Programme in the higher group while 15 out of 17 students were from the General Programme in the lower group. This is common when comparing students' achievement in mathematics and this result is expected. The students who are in the science classes are usually the high achievers while those who are not in the science class typically falls in the average or low achieving group. However, the result is alarming when those in the higher group have marks of 50% and less. Those students should have scored more than 50% since the science students are considered as high achievers in the overall Year 9 class level in the participating school. The two classes involved in this test were taught by the same teacher and also taught using the same method of teaching. The students' achievement in the SPA results is also reflected with the results of this test.

#### 5. Discussions

In order to improve the students' performance on the test, the teacher should first assess the students' prior knowledge especially with integers and algebra (Yahya & Shahrill, 2015; Matzin & Shahrill, 2015). A pre-test prior to the lesson can help the teacher to find out the students' prior knowledge. If the students have mastered their basics, the teacher could then proceed with the lesson on inequalities. According to Nebesniak (2013), in addition to focusing on procedures and computation, a teacher should include conceptual understanding where previously learned behaviours are confronted while encouraging students' understanding and ability to think mathematically. In this case, to consistently solve inequalities correctly, students need to understand the big idea behind the rule. An example given by Nebesniak was to begin the lesson with a true statement -4 < 6, graphed the two numbers on a number line and discussed what would happen to the statement and the graph if a positive number were added to both sides of the inequality. The discussion continued by adding a negative number to both sides and subtract positive and negative number on both sides. The final stage was to multiply and divide with positive and negative number to both sides and to discuss what happened to the inequality sign. Through scaffold inquiry, students discovered what happens to an inequality symbol when an inequality is multiplied or divided. This example explains the concepts of changing inequality sign and also how the numbers changed on a number line when all the operations were used.

Another issue found from the findings of the test was the difficulty in drawing and shading to represent the given inequalities. Again, the teacher should focus on the students' prior knowledge on drawing linear graphs before connecting the concept with inequalities. Students can draw graphs of simple inequalities but they often struggle when the format of the inequality is unfamiliar. When they produce a correct graph of an inequality, they still lack a deep understanding of the relationship between the inequality and its graph (Switzer, 2014). Typically, in teaching students the topic on inequalities, most students were observed able to shade inequalities for horizontal and vertical lines but not when the lines have negative or positive slopes. An example of an effective instruction mentioned by Switzer is by graphing with symbols. Students were given two linear expressions to begin with. The students were asked to choose three ordered pairs (x, y) between -10 and 10 to evaluate the two given linear expressions. From the calculated values, students wrote the symbols =, > or < between the two expressions to see the connection between the value of each expression, the relationship between the expression and the mathematical symbol used to represent that relationship. The students then graphed the pairs on a Cartesian plane. After they had plotted the points, students were able to see that the equal signs were plotted along a line, the < signs were on one side of the line and the > signs were on the other side of the line. The students made this discovery themselves and discussed why the arrangement had occurred.

# 6. Conclusions

Even though the test items were not piloted for its reliability, the subject teacher approved the questions to be used in the test for validity. However, the subject teacher commented that the questions were easy and students should be able to

answer most of the questions. The teacher also said that the questions were straightforward and would stimulate students to think and identify between big numbers and small numbers. For example, questions 15 and 16 tested the students' application skills on drawing straight-line graph as well as identifying the region represented by the given inequalities. However, the test items were found to be very difficult for the students. An average mark of 22% together with the percentage difficulty for every item has proven that the test was difficult.

The purpose of this test items analysis was to explore if the students understood the topic on inequalities that the teacher has taught recently. It is clear that the students have not understood or was unable to retain the information. Additionally, based on the few general findings from previous studies conducted in the context of mathematics education in the nation, students mainly find mathematics difficult because they do not understand the mathematical concepts in the first place (Botty et al., 2015; Botty & Shahrill, 2014; Kani & Shahrill, 2015; Law et al., 2015; Pungut & Shahrill, 2014; Shahrill, 2005, 2009, 2011, 2013). Consequently, the next step should be giving the students the feedback from the test and revisit those subtopics that the students are found to be weak at as well as in focusing on the common errors and misconceptions that may arise (Sarwadi & Shahrill, 2014). With the suggested examples of instructions mentioned earlier, hopefully the students will be able to understand the topic and have the confidence to answer those questions that were left unanswered.

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## Appendix A

Topic Test: Inequalities

Name:	
Class:	Date:

#### Instructions:

- a. Please write your name, class and today's date above.
- b. Show your working clearly in the spaces provided and underline your answers.
- c. Answer **ALL** the questions.
- d. Do NOT use calculators.
- 1. In the box, represents an inequality sign "<" or ">".

8×5

2. In the box, represents an inequality sign "<" or ">".

 $4 \times (-3)$   $7 \times (-3)$ 

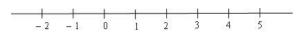
 $2 \times 5$ 

3. On the number line, mark the point x = 3.



If x > 3, draw an arrow the direction that best describes the inequality.

4. On the number line, mark the point x = -1.



If x < -1, draw an arrow the direction that best describes the inequality.

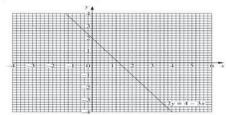
- 5. Solve the inequality  $3-5x \ge 18$ .
- 6. Solve the inequality 5x-4 < 28+x
- 7. Solve the inequality  $-\frac{2}{3}x \le 4$ .
- 8. Solve the inequality  $\frac{x+3}{2} > 4$
- 9. Solve the inequality -5 < 2x + 3 < 1
- 10. Write down all the integer values of x for which -3 < x < 1.
- 11. Find the integer values of x which satisfy  $-4 \le 3x + 2 < 8$ .
- 12. Find all the integers which satisfy both 2x+7<3 and  $x \ge -4$ .

13. Given that  $-3 \le x \le 2$  and  $-7 \le y \le 3$ , find a. the largest value of x-y and

- b. the smallest value of  $x^2$
- 14. Given that  $-5 \le x \le 3$  and  $-3 \le y \le 5$ , find
  - a. the smallest value of x+y and
  - b. the largest value of  $x^2 + y^2$
  - c. the value of x when  $x^2 = 16$ .

15.

The diagram below shows the line 2y = 4 - 3x.



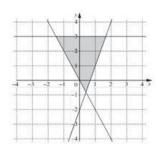
On this diagram.

(a) draw the line  $y = \frac{1}{2}x - 2$ ,

(b) shade and label the region, R, defined by the following inequalities.

 $x \approx 0$  $2y \le 4 - 3x$  $y \Rightarrow \frac{1}{2}x - 2$ 

16.



The shaded region on the diagram is represented by three inequalities.

One of these is  $y \ge 3x - 2$ .

Write down the other two inequalities.