Exploring 'Non-Science' Grade 11 Learners' Errors in Solving Quadratic Equations[1]

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Abstract

This study investigates the errors that a class of grade 11 learners show when they solve quadratic equations tasks through factorisation. The research makes use of the constructivist perspective of learning to explain learners' errors and misconceptions. The research took place at a high school in the East Rand of Gauteng Province, South Africa. Twenty two participants of both sexes regarded as below-average in mathematics performance were presented with four quadratic equations tasks which they were asked to solve. Learners' scripts were scrutinised for errors. Selected learners were then probed and pressed about their errors so as to manifest their thinking thereof. Analysis of data indicated that most students' errors arose from problems with factorisation; the pathway to solution. Some of the reasons why learners had errors are that they used inappropriate schemas to mediate their solutions. In the main, learners held on to the simple equation schema which they unsuccessfully used to assimilate solutions to quadratic equations when restructuring the schema was the only viable pathway. Also, most errors were due to mis-interpretation of what the tasks required. The study recommends that this topic may be taught using procedures with connections tasks (Stein, Smith, Henningsen, & Silver, 2000) to help students understand what they are doing.

Keywords: *errors, misconceptions, factorisation, equations*

1. Background to the Study

This study is about the errors and misconceptions that *'below average',* non-science Grade 11 learners have when they solve quadratic equations tasks through factorisation. It was undertaken by a pair of researchers; one based at a School of Education at the University of Witwatersrand, Johannesburg, South Africa and the other who is a mathematics teacher at an urban school in the same city.

One of the major goals of mathematics education in the new South African Curriculum Assessment Policy Statement (Department of Basic Education, 2012). 2012) is for learners to be able to investigate and solve problems in a creative manner (DBE, 2011, p. 9). This implies that in Grade 11, learners should be able to solve problems such as those involving problems in algebra effectively. Teachers play a key role in making this possible, but there are challenges learners encounter in making this possible. There are many challenges in teaching and learning of mathematics. This study concentrates on learner errors and misconceptions on quadratic equations at grade 11.

Our experience of teaching high school maths has enabled us to notice some patterns in the mathematical errors and conceptions that learners have, particularly in factorising algebraic expressions and equations. The errors Grade 11 learners make are similar to the errors that Grade 10 learners make. We have realised that even though learners are taught to factorise in Grade 9 and Grade 10 they still struggle to factorise in Grade 11. We have wondered; Why are they still not mastering the notion of factorising? We have realised that that learners have problematic schemas on these tasks. We argue that until we know why learners hold these errors, we will not be able to help them much.

Learners who leave high school to further study Mathematics in higher institutions of learning should have a good understanding of algebraic concepts. In addition, learners should also be able to apply these concepts correctly. This is

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what makes algebra different from other mathematics topics; it is the variables that seem to cause problems for learners. More so that they turn to create rules of their own that only works for them (Erlwanger,1975). Hence, errors and misconceptions develop as early as Grade 9 and they carry them through to Grade 11.

It is a fact that learners do make errors in learning mathematics, but what is an error? Past research was conducted on learner's errors and why are they making them (see for example White, 2005; Luneta & Makonye, 2010). Furthermore Olivier (1989) argues that "errors are the symptoms of the underlying conceptual structures that are the cause of errors" (p.3). The errors and misconceptions that Grade 11 learners make have been carried over from a lack of understanding encountered in the lower grades. Of course there are other conditions that might lead to learners having misconceptions and making errors. This study will concentrate on the cognitive factors related to learners' answering to mathematics tasks.

1.1 Problem statement

It is our experience that no matter how much we teach learners to factorise they still cannot seem to get the concept and understand it fully. Factorising in high school mathematics is a key technique serving as one of the pillars required to process mathematics. Furthermore (Luneta & Makonye, 2011) found out that there are many undergraduate mathematics students who hold various misconceptions on basic mathematics affecting the way they answer particle mechanics problems. This implies that if errors that learners show in earlier grades are not discovered and dealt with, learners transfer them to higher education.

1.2 Purpose statement and research questions

The study investigated the types of errors and misconceptions that below average grade 11 learners have in algebraic factorisation needed to solve quadratic equations. Then it explains why these learners still cannot master factorisation a procedure they could still not master in grade 10 and 9.

The research questions are: 1) What errors do the below average grade 11 learners show when they factorise quadratic expressions to solve quadratic equations? 2) What learner thinking is responsible for their errors in solving quadratic equations?

1.3 Rationale

Errors and misconceptions will always be there, since errors and misconceptions are part of learning and learners are learning all the time. This means that teachers must always research on learner errors. Learners in grade 11 are still not able to factorise quadratics although the skill of factorising is taught earlier in grade 9 and 10. That is a problem. Factorisation is a basic skill that one requires in order to be able to cope with mathematics related courses later in further education. Yet learners exit schooling without this skill. This creates some sort of a backlog where a student cannot catch-up has to always go a few steps back to learn the basics at the same time struggling with the current work at that particular course at a high institution of learning.

A study as this can help teachers in their professional development with an in-depth analysis on how to diagnose learner difficulties in a specific mathematics topic. This helps teachers to better do their work as they will be informed of specific learner difficulties such the errors they are prone to and what schemas influence their occurrence.

2. Literature Review and Conceptual Framework

2.1 Conceptual framework

This study is informed by constructivist theory of learning. The constructivist theory of learning imply that learners come to a new grade not as empty vessels but they come with some pre-knowledge acquired in the previous grades, which knowledge they use to assimilate and adapt incoming mathematical concepts (Hatano, 1996; Olivier, 1989). So the new knowledge they learn interacts with their prior knowledge and learners try to find the balance; to equilibrate between what they know already and what they are learning now. The process of finding the balance between prior and current knowledge may lead to errors. This occurs for example when a learner uses an unmodified earlier acquired schema to mediate new knowledge when in fact that schema is not appropriate for the extended domain. From constructivist point of view misconceptions plays important role in teaching and learning since they are the way that learners interact with new mathematical concepts to make sense of them (Brodie, 2005). If learners mechanically use old schema without adapting them to the new situation they make misconceptions due to instrumental rather than relational understanding (Skemp, 1976).

2.2 Literature review

Learners who produce errors systematically find it hard to accept *correct instructions* that counter what learners thought was right all this time. This is consistent with the findings of Smith, DiSessa & Roschelle (1993) who have argued that leaners are reluctant to give up their misconceptions because they have meaning to them and they have constructed them themselves. Giving them up is similar to someone destroying a house they have built.

For example in lower grades learners learnt algebra by filling in the numeral in the place holder.

3 + = 7 .

Here, learners knew that they have to find a value when you added it 3 that will result in 7. In the higher grades, learners find it difficult to understand that the place holder can have a coefficient. For example: $3 + \frac{1}{2}x = 7$. The coefficient affects the place holder value and that needs to be taken into account.

Errors are rooted in some mathematical concept that learners learnt in their earlier grades. Nesher (1987) argues that errors originate from earlier acquired valid mathematical knowledge. For example; in earlier grades learners learnt that number 5 is bigger than number 2. They find it difficult to understand that -2 is greater than -5.

We need to accommodate learners making errors. “Misconceptions are crucially important to learning and teaching because they form part of a learners’ conceptual structure” (Olivier, 1989, p.3). We as teachers should not think that misconceptions are a bad thing but instead we should use them to our advantage. For example; to use them as a reflection tool to our own teaching by finding a different way of teaching a mathematical concept based on the learners’ misconceptions.

Erlwanger (1973) states that: “misconceptions can sometimes work positively in favour of the learners” (p.45). The example of a learner named Benny who continually made errors and had misconceptions agrees with Erlwanger’s statement. Benny was not aware of the fact that he had misconceptions about mathematical concepts simply because he was getting correct answers and even his teacher could not pick that up. From the story of Erlwanger it is evident that if errors and misconceptions are not picked up earlier, the learners will continue making more errors and creating their own wrong mathematical rules.

2.2.1 Some types of errors

Research into misconceptions in algebra is huge and widespread; there are many reports as to why errors and misconceptions occur and differing ways to classify them. Clark (1973) classifies errors into three types namely: operator, applicability and execution.

Operator errors are evident on learners who often reflect incorrect knowledge. For example in the equation:

Factorise the following:

$x^2 + 3x - 6 = 0$

$(x + 5)(x - 1) = 0$

This basically shows that this particular learner has not grasped completely the concept of factoring a quadratic expression. Here he/she has not included the middle term in the factors; that on its own reflects an incomplete knowledge.

An *applicability error* involves the misuse of the rules of algebra. For example:

$3(2a + 1) = 5a + 6$

In the above example the distributive law is applied incorrectly in this instance. Instead of multiplying into the terms in the brackets this learner somehow added the coefficient of a with the numeral outside the brackets.

Execution errors include partial executions, such as;

$2(x + 1) = 2x + 1$

Sometimes learners do copy the problem incorrectly.

3. Research Methods

This research uses qualitative rather than quantitative method. The quantitative method of research is mainly used when a researcher uses measurements to yield statistical data to prove or disprove a prior hypothesis stipulating relationships between variables of interest (Rolfe, (2006). However, the qualitative method refers to emphasis on meanings attributed to social phenomena such as those affecting teaching and learning. Qualitative research mainly focuses on its in-depth description of experiences of the participants and their subjective views. This study is about the errors and misconceptions that learners show in the classroom (Cohen, Manion & Morrison, 2002). This phenomenon is best understood through qualitative approaches rather than quantitative, as the researchers play an important role in interpreting the data they obtain in the light of the research questions of interest.

3.1 Sampling

A class of 22 below average male and female Grade 11 learners of different ethnicity aged between 16 and 18 years old was chosen as a sample for the study. The study was conducted at a former model C school in an urban town east of Gauteng Province. One of the researchers taught this class. The sample class was the bottom end of three classes at the school in terms of performance. There are learners who got between 30% to 40% marks in Grade 10. For this reason this class does not study science subjects as they are deemed too difficult for them. None of the learners were repeating Grade 11, meaning that all of these learners in the class are meeting Grade 11 level quadratic equations for the first time. We chose this particular class simply because they are not doing particularly well in mathematics and hoped that it will provide us interesting data.

3.2 Data collection methods

Data was collected from learners in an accommodative environment which was their normal classroom. This helped learners not to feel at ease which would affect the good reliability of the data collected.

Participants were given a mathematical tasks which had four problems to be answered. Their scripts were marked attentively looking for errors and misconceptions that learners may have on the tasks. Based on the answers they wrote in the tasks; interviews then followed on a one on one basis for thirty minutes. Since we could not fully tell what the learners were thinking through their written answers, we felt that the best way was to get it from 'the horse's mouth'. The interviews helped us to get an in-depth understanding of why learners made the errors they made and the root of their misconceptions.

3.3 Rigour

According to Golafshani (2001) reliability is a concept that measures and evaluates the quality of the study concerned, and to the extent that results of the study are indeed consistent over time (Golafshani, 2003). Golafshani (2003) argues that validity determines whether the study has measured what it has intended to measure or how truthful are the results of the same study. In other words does the research methods help in getting the responses that answer the research questions. Also, utmost respect was accorded to learners. We encouraged participants to freely express themselves and assured them that the research had nothing to do with their school grades. We focused on examining the error types from the scripts and obtaining the reasoning behind the errors through probing learners on them so that they would manifest their reasons. In this way we believe rigour was ensured.

All ethical considerations were adhered to. Confidentiality and anonymity were guaranteed to participants. The parents of the learners were given the information letter about the research and they gave consent for their children to participate. Only learners whose parents consented participated in the study.

4. Data Analysis

The grade 11 class wrote tasks consisting of four quadratic equations. These are:

Level 1 problems: $(x - 5)(x - 2) = 0$ and $x^2 + 5x + 6 = 0$

The problems in this level required learners to immediately factorise and solve for the correct value of the unknown. For the problem given in a factor form they only had to find values of the unknown.

Level 2 problem: $x^2 + 2x - 3 = 12$

This problem was not given in a standard form. Meaning learners had to do a little bit of manipulation before they could solve the problem. The first step was to write the equation in a standard form to make it level 1.

Level 3 problem: $x(x + 1) = 6$

This is a higher level problem. Learners had to use distributive property of algebra correctly. Then transpose 6 to the left hand side of the equation to get the equation into a standard form.

4.1 Conceptual framework used: Different categories of errors

- *Systematic errors*: According to Cox, (1975) "an error is labelled systematic when there is a repeatedly occurring incorrect response that is evident in a specific algorithmic computation" (p. 203).
  Example: $x^2 + 2x - 15 = 0$ factors $(x + 3)(x - 5) = 0$ it is systematic since a learner would only find factors for the first and last term and ignore the middle term.
- *Random errors*: Cox (1975), argue that random errors are made by the learner who basically knows how to work out the problem and going through processes to solve the problem. No pattern of incorrect systematic errors is evident in the working out.
  Example: Learners failing to get the correct value of the unknown by forgetting to change sign of the content when transposing it. If given $x + 2 = 0$ learner will write $x = 2$ instead of $x = -2$
- *Conceptual errors:* these are errors that occur when learners completely misunderstood the concept and how it is applied.
  Example: A learner will falsely factorise a quadratic equation that is not written in standard form. $x^2 + 2x = -1$, then $x^2 = -2x -1 \therefore x = -2$ divided by x both sides but a learner ignored the -1.
- *Procedural errors:* misapplication of the mathematical rule.
  Example: $x(x - 6) = x^2 - 6$

4.2 Analysis of written for error types

4.2.1 The analysis of learners' responses on problem 1

The vignette below Fig. 1, is an exemplar response from one learner who shows an incorrect order of operation. I call it an operation because to be able to get correct factors, as a learner you might have gone through some operations. For example checking your answer by multiplying-out factors and getting the equation you initially started with. More so that if your factors are not correct, it means the operation that a learner used is incorrect.

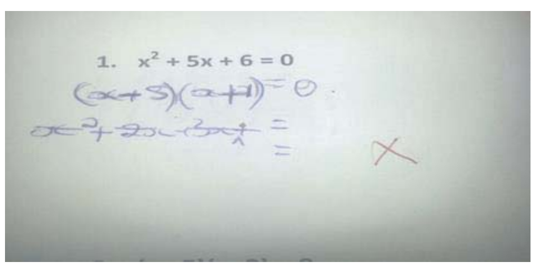

Figure 1

Factorising a quadratic equation requires learners to take into consideration all other three terms at the same time. For example the factors of the first term and the last term in the equation added together or sometime subtracted must work-out to the middle term. The learner in this example chose the incorrect terms of the last term because five times one equals to five not six. The learners worked from the 'inside - out' used the factors of five the middle term (5 and 1) and added them to get the last term 6. The learner got the concept of factorising completely messed up. Learner knows what to be done, but not in a sequential manner. Learners who show the same error pattern of this nature, knows that they have to find the factors of the two other terms and then manipulate them to get the middle term. They do not know or confuse which of the terms to first find their factors.

The Fig. 2 below shows the incorrect use of the quadratic formula. Correct substitution of the variables into the formula, but wrote down the quadratic formula incorrectly. The learner had '$\mathbf{b^2+ 4ac}$' inside the square root instead of the correct '$\mathbf{b^2 - 4ac}$' thus getting the wrong values of the unknowns.

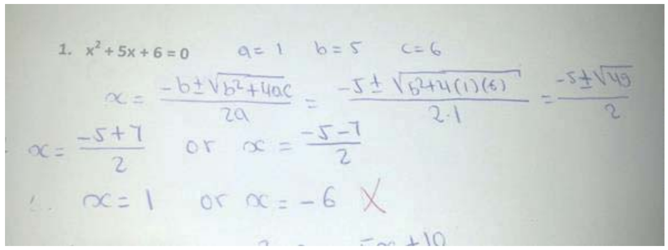

Figure 2

4.2.2 The analysis of learners' responses on problem 2

The following in Fig.3 below is a typical example of a learner who does not understand that quadratic equations can be represented in different forms. Problem 2 is a quadratic equation given as factors. All the hard work has been done for the learner! All she/he has to do is just to equate the factors to zero then find the values of the unknown.

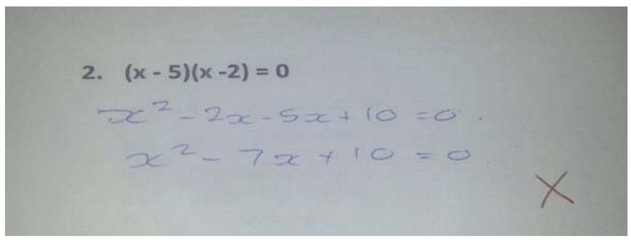

Figure 3

This learners' thinking is that the problem still has to be worked on it. Failing to notice that it written in factors form, instead the learner multiplied out the brackets correctly but that was not what he/she needed to do. This learner did not read the question to understand it, because the question was to find the value of the unknown and the learner did not work towards that. Reading to understand what the question asks of is one of the challenges that learners have to overcome. It is evident in figure 3 above that the learner did the correct mathematics but got stuck at the end because she/he did not know what to do next. The issue here seems not to be mathematics, but reading to understand what is required and then apply the mathematics required to solve the problem.

4.2.3 The analysis of learners' responses on problem 3

4.3 Transformations

Some of the problems required learners to do some transformations, move the term(s) to the left hand side of the equation and write the equation in a standard form. In this problem the transformation skill had to be applied before any further calculations can be done.

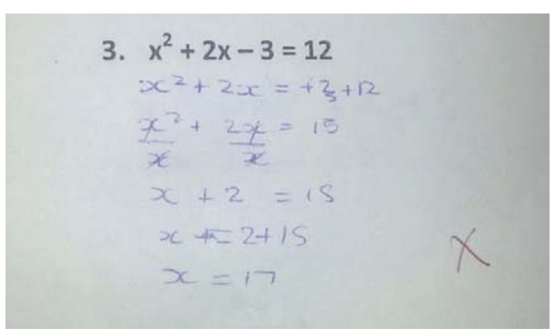

Figure 4

In the figure 4 above is the example of a learner who shows applicability error of the transpose skill thus the failure to correctly complete the problem. The learner went further on to show errors by only dividing the right hand side. This shows the lack of understanding of the equals sign. The left hand side is equals to the right hand side of the equation, meaning any changes you do on any of the two sides must be done on the other side as well. The learner knew that at the end she/he had to have a linear equation then find the value of the unknown. But this learner went all wrong to get his/her linear equation, dividing by x must have been on both sides of the equals sign. Further in the second last step of the learners' response in figure 4 above, transformations error occurred. 2 were transformed to the right hand side but it kept the same sign it had in the left hand side.

Another illustration of an error pattern is the applicability of the distribution property as shown in figure 5 below. The learner misinterpreted and used the rule incorrectly. The distributive property is applicable only when the terms that are distributed are in the same side of the equals sign.

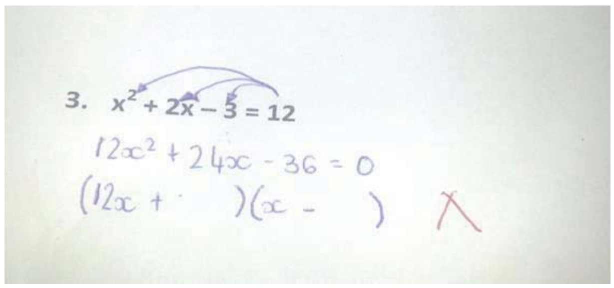

Figure 5

4.3.1 The analysis of learners' responses on problem 4

The common error in this problem is the failure to distribute all the way to parentheses. Learners somehow are like 'half distributing' for example distributing one term in the brackets and not multiplying the other term.

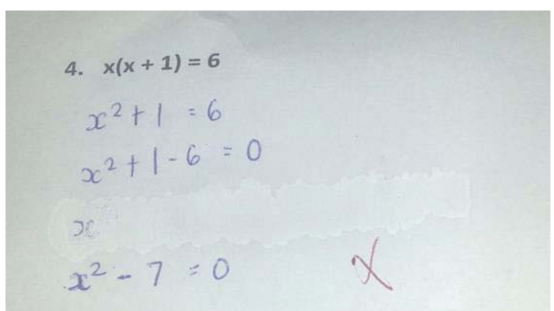

Figure 6

The learner in Fig. 6 only distributed to the first term 'x' in the brackets and somehow did not do the same thing to the second term '1'. This learner did the transformations skill correctly moving the 6 to the left hand side and changed the sign to negative 6. Then collected the like terms incorrectly added '**1 - 6**' to get '**-7**'.

Table 1: Comparisons of the Percentages of the Three Categories of Errors in the four Problems in the Task

Level of the problem	Conceptual and application errors	Arbitrary errors	No errors
1	29, 4%	17, 7%	52, 9%
2	47, 1%	0%	52, 9%
3	35, 3%	29, 4%	35, 3%

Table 1 shows the percentages of learners who showed the type of an error in each of the four problems, where we combined problem 1 and problem 2 under problem 1. The problems might look different how they are written up, but the processes involved solving the two problems is the same. For that reason we grouped them together under problem 1 in table 1 above.

4.3.2 Level 1 problem

4.3.2.1 Systematic conceptual and procedural errors

29,4% of the learners showed systematic conceptual and procedural errors when working out level 1 problem. These are learners showed a lack of knowledge on factorising quadratic expressions. In this category some learners failed even to get the first step correct. That is failing to notice the different forms the equations can take, therefore struggling to further solve the problem. In addition learners also failed to get the correct factors of the given quadratic equation. They did not have the basic concept of factoring for example some wrote (5 and 1) as the factors of 6, thus getting the factors quadratic equations all wrong. We also realised that all the learners who showed systematic conceptual and application errors in the level 1 problem, went to commit the same errors in the follow on problems.

4.3.3 Arbitrary errors

These learners changed the questions to a form simpler to them. For example instead of having 'x^2**-7**x + 10' they will have 'x^2**+7**x + 10'. These errors also involved changing a quadratic equation to a simple equation.

4.3.4 Level 2 problem

4.3.4.1 Systematic conceptual and procedural errors

The percentage of the systematic conceptual and procedural errors went up from problem 1 to problem 2 by 17, 7%. That can only mean more learners seem to have difficulty when they have to do more than one operation in order to solve the problem. In this problem, learners had to transpose one term and write the equation in a standard form then factorise. Some learners did not know what to do with the 12 in the right hand side of the equation. Those who knew that they had to transpose, did apply the transpose rule incorrectly by not writing the equation in standard form.

4.3.5 Level 3 problem

4.3.5.1 Systematic conceptual and procedural errors

The common bugs in this problem is that of the incorrect order of operations; such as using the distributive property even on the terms that are not in the same side of the equals sign. This is a typical example of learners who show instrumental understanding of the distributive rule. Learners had to perform more than one operation to solve the problem, and learners were getting confused on the order of the operations on which one to do first. This only happens when learners have partial understanding of the factorising procedure; they do not understand; they rely on a schema that is not refined for this.

4.4 Analysis of interviews

We choose the extract below, Fig. 7, as representative of the interviews as it shows how a learner gets it completely wrong; way before the learner can apply mathematics to solve problems. Here the learner does not understand what the question requires them to do. The extract below is of the learner who wrote the following response;

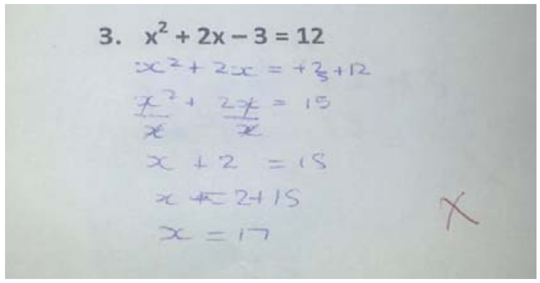

Figure 7

Researcher	Please read the question to me.
Learner A	solve the following and show all working
Researcher	…what is the question asking you to do?
Learner A	asking me to solve the equations, I guess
Researcher	What do you mean when you say "I guess" do you exactly know what you must do?
Learner A	of course I know sir
Researcher	ok take me through on how you got to the answer
Learner A	**I have to make x the subject of the formula and then find its value**
Researcher	is it a value of x or values of x?
Learner A	Hmmm it is values… *pause*… nope it is the value of x sir, that is why I divided by x on this side (LHS). I then moved 2 to the other side (RHS) solved for x equals to 17

This expert shows that the leaner had a misconception as he/she was using the simple equation schema to mediate solution to the question in an instance where it was not suitable to apply it. He/she was assimilating a new mathematical object into a prior inappropriate schema; a case of putting new wine in old bottles if one may say. What was required was to grow a new schema to accommodate a completely new concept of solving quadratic equations by factorisation without however discarding the simple equation solving schema, as it is a subset of the quadratic equation solving cognitive structure.

In the interviews conducted, participants were consistent in terms of the errors and misconceptions they showed on their written responses as well as the answers they gave in the interviews. We report here that some learners later realised their errors and misconceptions as we probed them to further explain their working. The probing helped participants reflect more deeply on their answers helping highlight their wrong thinking. This lead to learner self-correction and misconceptions resolution.

5. Discussions

What we picked up from the study is that participants showed many errors in their responses even to one task (see vignettes for problem 3 and 4). This is consistent with Makonye and Luneta (2014) finding, that learners make different errors in solving even one mathematics task. It is not that learners do not know mathematics; rather they confuse the concepts and are not clear on which one to use and when it can be used. This entanglement of learners with many different concepts and ideas of mathematics is one reason why learners find mathematics difficult. They cannot sort out which schemas to apply and which not to apply. Often they assimilate new concepts into old schemas in instances where they are supposed to refine their schemas; to accommodate and grow their schemas so that they can capture new information. We have found that learners hold on to old schemas instead of growing them. This results in many misconceptions. Many of the systematic errors made by learners were due to the lack of understanding of the need to build new schema. In most cases learners did not know what to do and struggled to fit new ideas onto limited prior ideas. Often learners displayed a lack of understanding what the question asked of them to do and interpreted the task to conform to what they already knew, such as converting a quadratic equation to a simple equation (see vignette 3).

6. Conclusions

The findings of the study provides an insight into types of errors grade 11 learners make when solving quadratic equations through factorisation and the possible causes in the South African scene.

7. Findings

7.1 Types of errors in solving of quadratic equation

Some of the errors were quite obvious but some were very difficult to classify. The obvious errors were easy to pick up since they were repeated over and over in all the problems by different participants. These are mainly systematic conceptual and procedural errors.

A further classification are:

- *Applicability errors.* Clark (1973) referred to these as misuse the rules of algebra. For example the incorrect use of the distributive rule in a problem that does not even require the use of the rule.
- *Procedural errors:* An example of these is *a transpose error* where learners fail to transpose correctly by changing the sign of the term when it transposed from one side to the other.
- *Conceptual errors: T*hese are errors where learners do not comprehend the ideas subsumed by the task, such as handling a quadratic equation as though it were a simple linear equation.

We rush to note that these error types are not at all comprehensive. Indeed the errors mentioned could be classified under different names depending on the taste of the researcher, but in our view these are the some of the important errors we discuss in the limited time we had.

7.1.1 What are the reasons for the errors?

Interviews with learner revealed that in most cases learners used an earlier and limited schema to mediate solution to quadratic equation tasks. For example learners retrieved the simple linear equation schema to solve quadratic equations. They failed to come up with the factorisation cognitive structure needed for this purpose. Where factorisation was used learners often failed to factorise correctly, catering mainly to satisfy two terms instead for all three so that on re-expansion we can have the original expression.. This showed learners' lack of conceptual understanding of the factorisation principle; lack of conservation of the original expression. Also it was found out that in some cases, learners do not read or understand instructions to the questions resulting in them failing to answer questions correctly. For example learners would expand brackets working backwards to come up with a quadratic expression there by hardening the question instead of just reading the answers. This error was due to the fact that learners earlier learnt how to expand binomials and so to them the binomials invoked to them need to expand them. So interpretation and understanding questions came up as one of the main reasons learners had errors. Learners answered questions they were never asked.

8. Recommendations

Teachers need to create environments in their classrooms that allow learners to freely express themselves so that limits to the prior knowledge learners have on certain mathematics concepts and procedures can be discussed and restructured. In order to improve teaching and learning of quadratic equations, it is important that teachers are always on the lookout for learners' errors and be in the position to make appropriate discussions on them so that learners may correct them. Importantly because the research shows that in the main students are dealing with procedures without connection to meanings, it is important that the topic of solution of quadratic equations be taught with procedures with connections to meanings (Stein, Smith, Henningsen, & Silver, 2000). This way the use of graphs is very important in this regard.

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