INNOVATIONS IN THE MATHEMATICS TEACHING IN SECONDARY SCHOOL

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Abstract

The subjects of research are concrete possibilities of use philosophical-historical-developed and methodological-innovative elements in the mathematics teaching in elementary and secondary school in breaking of formalism and activating e.g. dynamism of teaching. The aim is that the pupils think over with originality and creation thinking. It develops usage of indepedent thinking, critical estimation and reasonable generalization.

Keywords: Innovations, philosophical-historical-developed elements in the mathematics teaching, methodological-innovations, breaking of formalism, dynamism of teaching mathematics.

1. Introduction

Properties of important points of a triangle (orthocenter, center of gravity, centers of a circumscribed circle and centers of an inscribed circle) as well as methods of their construction have been known since ancient times. However, it is less known about the existence, properties and structures of some unusual points of a triangle, such as J. Steiner's (1796 - 1863) point (which has the property that the sum of the distance from it to the apex of a triangle is minimal), or H. Brocard's (1845 - 1922) point (with the property $\measuredangle \Omega$ AB=$\measuredangle \Omega$ BC=$\measuredangle \Omega$ CA where Ω is the inner point of triangle *ABC*), or of some other less important points. [1]

On this occasion we will not deal with the well-known properties and structures of J. Steiner's point, nor with the properties and structures of Brocard's point of a triangle, but with properties and constructions of a less distinctive and less familiar point of a triangle for which the following theorem is true.

2. Properties of an Unusual Point of a Triangle

Theorem 1. If three congruent circumferences intersecting in point S_1 and each touching two sides of ΔABC are inscribed in the triangle and if points O and S are the centers of the circumscribed and inscribed circles of ΔABC, then points O, S and S_1 are collinear.

Proof:

Let us designate with points A_1, B_1 and C_1 centers of the given congruent circumferences. It is easy to notice that they belong to the bisectors of the interior angles in ΔABC (see figure 1). $A_1L = B_1G \Rightarrow \overline{A_1B_1} \parallel \overline{AB}.$ In the similar way, we conclude that $\overline{A_1C_1} \parallel \overline{AC}$ and $\overline{B_1C_1} \parallel \overline{BC},$ from which follows that ΔABC and $\Delta A_1B_1C_1$ have common bisectors of the interior angles, that is they have one common center S of the inscribed circles. It is easy to note that a homothety H with center S and coefficient k maps $\Delta A_1B_1C_1$ into ΔABC, i.e. $H_S^k(\Delta A_1B_1C_1) = \Delta ABC,$ where $k = \dfrac{AB}{A_1B_1}.$ However, since we have that, according to the given problem, $S_1A_1 = S_1B_1 = S_1C_1$, point S_1 is the center of the circumscribed circle around $\Delta A_1B_1C_1.$ Since ΔABC and $\Delta A_1B_1C_1$ are homothetic, the circles circumscribed around them are also homothetic, therefore we have that $H_S^k(S_1) = O,$ i.e. points $O,$ S_1 and S are collinear, which was to be proven.

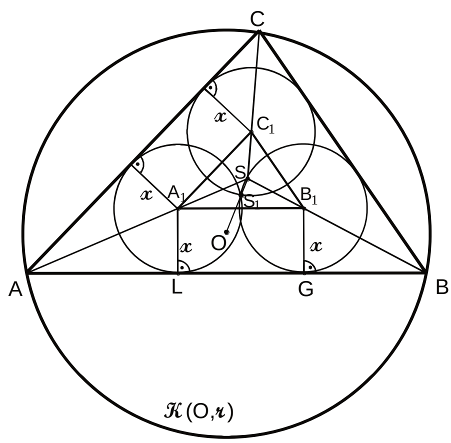


Figure 1.

Property of colinearity of points $O,$ S_1 and S enables us to construct point S_1, that is to solve the following problem.

3. Constructions of an unusual point of a triangle

2. Inscribe (construct) in the given triangle ABC three congruent circumferences intersecting in point S_1, where each circumference touches two sides of the triangle.

Analysis: According to the previous theorem, we have that a homothety H with center S and coefficient k maps $\Delta A_1B_1C_1$ into ΔABC, i.e. $H_S^k(\Delta A_1B_1C_1) = \Delta ABC,$ where $k = \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} = \frac{SA}{SA_1} = \frac{SO}{SS_1}$ (see figure 1).

In order to construct the requested point S_1 of the given triangle it is necessary and sufficient to find the homothety (similarity) coeficient.

For that purpose, we construct a figure similar (homothetic) to the one in picture 1. That figure given in picture 2 consists of a pair of homothetic triangles $\Delta A_0B_0C_0$ and $\Delta A_2B_2C_2$ which due to similarity to the figure in picture 1, i.e. similarity to a corresponding pair of homothetic triangles ΔABC and $\Delta A_1B_1C_1$ have the same homothety coefficient k.

We obtain this figure by constructing in the following way:

First let us construct any $\Delta A_0B_0C_0$ which is similar to a pair of homothetic triangles ΔABC and $\Delta A_1B_1C_1$, and which will concurrently be homothetic with triangle $A_2B_2C_2$, where the center of that homothety is point S_0, and coefficient k, i.e. $H_{S_0}^k(\Delta A_0B_0C_0) = \Delta A_2B_2C_2$.

Let us construct centers of inscribed and circumscribed circles in and around $\Delta A_0B_0C_0$, that is let us construct bisectors of its interior angles in whose intersection is found point S_0 - the center of the circle inscribed in $\Delta A_0B_0C_0$, and let us construct bisectors of its sides in whose intersection is found point S_2 - the center of its circumscribed circle (see figure 2). Let us construct three congruent circumferences which have one common point S_2 (the center of the circumscribed circle around $\Delta A_0B_0C_0$), with centers A_0, B_0 and C_0 and with equal radiuses $A_0S_2 = B_0S_2 = C_0S_2 = y$. It is easy to construct common tangents (A_2B_2), (A_2C_2) and (B_2C_2) on pairs of those circumferences that generate triangle $A_2B_2C_2$ homothetic with $\Delta A_0B_0C_0$.

Since the homothetic figure in picture 2 is constructed in the way that it is similar to the homothetic figure in picture 1, it means that the homothety coefficent is.

$$k = \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} = \frac{SA}{SA_1} = \frac{SO}{SS_1} = \frac{A_2B_2}{A_0B_0} = \frac{B_2C_2}{B_0C_0} = \frac{C_2A_2}{C_0A_0} = \frac{S_0A_2}{S_0A_0} = \frac{S_0O_0}{S_0S_2}$$

Construction (first way):

Through analysis we found that a pair of homothetic triangles ($\Delta A_0B_0C_0$ and $\Delta A_2B_2C_2$) of the figure in picture 2 has the homothety coefficient that equals the one of a pair of corresponding homothetic triangles (ΔABC and $\Delta A_1B_1C_1$) of the figure in picture 1, i.e.

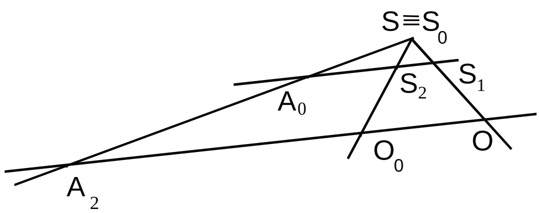


Figure 3.

$k=\frac{S_0A_2}{S_0A_0}=\frac{S_0O_0}{S_0S_2}=\frac{S_0O}{SS_1}$, therefore we can easily construct the requested segment S_0S_1 .

Let us observe a part of the construction in figure 2, precisely let us construct a pair of corresponding homothetic triangles separately ($\Delta S_0A_2O_0$ and $\Delta S_0A_0S_2$) (see figure 3).

Triangle ABC is given and we can easily construct points S (the intersection of bisectors of its interior angles) and O (the intersection of bisectors of its sides) (see figure 4). Homothety H with center S and coefficient k maps $\Delta A_1B_1C_1$ into ΔABC , i.e. $H_S^k(\Delta A_1B_1C_1)=\Delta ABC,$ where $k=\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}=\frac{SA}{SA_1}=\frac{SO}{SS_1}.$

Therefore, in figure 4, point S_1 is to be constructed, that is the segment $SS_1=S_0S_1$. As we constructed a segment SO and since $k=\frac{SO}{SS_1}$, we can circumscribe in figure 3 a circle K (S_0, S_0O) with a center in the homothety center $S\equiv S_0$ and a radius $S_0O=SO$. With this homothety, points A_0, S_2, S_1 match points A_2, O_0, O , i.e. $H_S^k(\Delta A_2O_0O)=\Delta A_0S_2S_1.$ Point S_1 is obtained as intersection of straight lines (S_0O) and (A_0S_2) , i.e. $S_1=(S_0O)\cap(A_0S_2)$. Thus we constructed segment $S_0S_1\cong SS_1$ and on segment SO we easily find the requested point S_1 of triangle $ABC.$ In the similar way we can construct vertexes of $\Delta A_1B_1C_1$, and thus the three congruent circumferences in ΔABC that have the common intersection in point S_1 (figure 4).

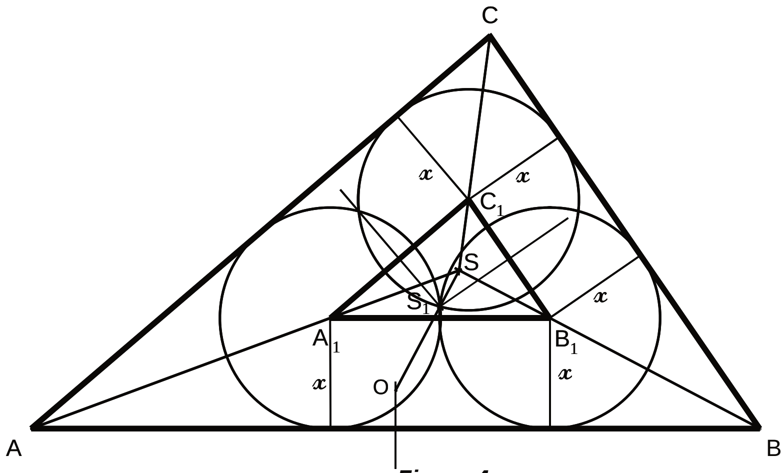


Figure 4.

Construction of the requested point could have been done in the following way as well:

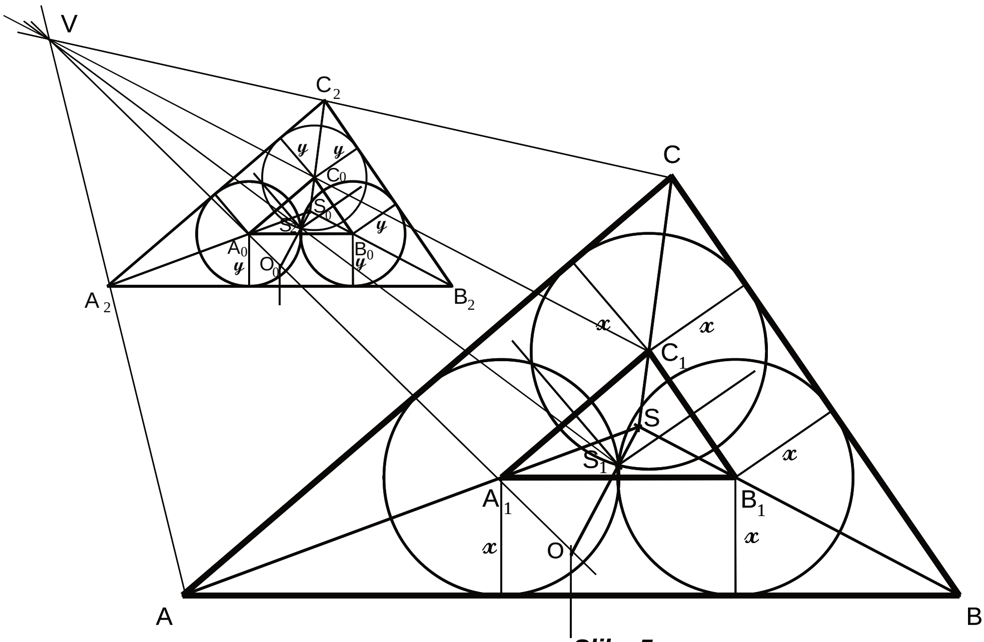
Let us construct again a pair of homothetic triangles ($\Delta A_0 B_0 C_0$ and $\Delta A_2 B_2 C_2$) of the figure as in picture 2, and then let us construct ΔABC in homothety with $\Delta A_2 B_2 C_2$. It is clear that in this case their homologous (corresponding) sides are parallel. Center *V* of a new homothety is found in intersection of pencils of straight lines $(AA_2) \cap (BB_2) \cap (CC_2) \cap (SS_0) = V$, where the coefficient of this homothety (similarity) is

$k_1 = \frac{VA}{VA_2} = \frac{VB}{VB_2} = \frac{VC}{VC_2} = \frac{VS}{VS_0}$, then we have $H_V^{k_1}(\Delta ABC) = \Delta A_2 B_2 C_2$ and $H_V^{k_1}(S_1) = S_2$.

Therefore, point S_1 is found in intersection of straight lines (VS_2) and (SO), i.e. $(VS_2) \cap (SO) = S_1$, and the construction of point S_1 is thus completed (see figure 5).

However, we have that $H_V^{k_1}(\Delta A_1 B_1 C_1) = \Delta A_0 B_0 C_0$, and:

$(VA_0) \cap (SA) = A_1$, $(VB_0) \cap (SB) = B_1$, $(VC_0) \cap (SC) = C_1$, therefore we can construct point S_1 in a third way as a center of a circle circumscribed around $\Delta A_1 B_1 C_1$.



Slika 5.

Proof:

Proof ensues from analysis and construction.

Discussion:

Since point S_1 as the center of the circumscribed circle around $\Delta A_1B_1C_1$ is single (only one), the problem has only one solution.

Such innovations presented in classes of teaching in high school result in students' increased interest in independent research or creative work and serve for eradicating formalism and activating or dynamazing the teaching process with talented students.

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