Elementary Teachers' Knowledge for Teaching Mathematics: A Review

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Abstract

The objective of this review is to provide a clear interpretation and implication of the research for policymakers and researchers who are interested in elementary teachers' knowledge for teaching mathematics. Based on the objectives, I examine 20 empirical studies in the area of elementary teachers' knowledge for teaching mathematics. The fundamental criterion for grouping studies is their research questions. The studies' research questions are related to each other to a greater or lesser detail. Thus, the research questions are categorized into four groups. The four groups are (a) how can we define elementary teachers' knowledge for teaching mathematics? (b) How can instruments for measuring elementary teachers' knowledge for teaching mathematics be developed? (c) What factors are to be related to elementary teachers' knowledge for teaching mathematics? And (d) what are international studies saying about elementary knowledge for teaching mathematics? For each group, description of all relevant studies and findings and interpretations are presented. The analysis of studies demonstrates that expanding the field of researchers will provide new ways of examining elementary teachers' knowledge for teaching mathematics and at the same time may provide critical information about affects and factors of knowledge for teaching mathematics.

Keywords: *pedgagogical content knowledge; knowledge for teaching mathematics; elementary teacher; teacher education; educational policy;*

1. Introduction

Since the No Child Left Behind (NCLB) act took effect in 2001, student achievement became a new indicator to show results of education (Dee & Jacob, 2011). For this reason, interest in various factors that might raise student achievement has increased. Most important, policymakers and national organizations begin to focus on how teachers might help promote students' high academic achievement (U.S. Department of Education, Office of Postsecondary Education & Office of Policy Planning and Innovation, 2002). In particular, recent research revealed that student achievement in mathematics is most easily affected by mathematics teacher quality (River & Sanders, 2002). Then, how can we define mathematics teacher quality?

There are studies that attempt to find the answer to this question concerning teachers' knowledge (e.g. Hill, Rowan & Ball, 2005). In particular, elementary teachers' knowledge for teaching mathematics is significant because teachers at this level more easily affect younger students' mathematics achievement scores than is the case for older students (Hill, 2008; Konstantopoulos, 2011). Thus, because it is viewed that elementary teachers' knowledge for teaching mathematics influence student achievement in mathematics, it is important to study the nature of this impact on student achievement. Furthermore, this understanding might inform education policymakers' decisions about how to improve elementary mathematics teacher education.

This review is a critical examination of empirical studies in the area of elementary teachers' knowledge for teaching mathematics in which to present the implications and implementation that emerge from the review. The literature review is divided into three major sections. In section one, I describe the methods employed, including the criteria for the studies included in this review. In the second section, I discuss findings and implications for four categories of questions based on elementary teachers' knowledge for teaching mathematics. These questions are used as a framework for this review, which consist of clusters of studies that are relevant to the questions. In addition, these studies are discussed in detail regarding their findings and implications. Finally, in section three, to provide a base for general implications for researchers and policymakers, I argue that the results of this review may provide pertinent information about how to develop pre-service or in-service teachers education programs to improve the preparation and professional development of elementary teachers' knowledge for teaching mathematics.

2. Methods

The objective of this review is to provide a clear interpretation and implication of the research for policymakers and researchers who are interested in elementary teachers' knowledge for teaching mathematics. Based on this objective, this section presents and discusses the framework for organizing the studies examined for this review.

2.1 Scope of the review

At the time of this review, 20 studies satisfied the criteria. The criteria for inclusion are as follows.

First, in order to focus on elementary teachers' knowledge for teaching mathematics, I discounted studies that compared elementary and secondary teachers' knowledge for teaching mathematics and studies that were conducted without distinction between elementary and secondary mathematics. Also, studies that looked at pre-service elementary teachers were excluded because elementary teachers knowledge for teaching mathematics might be related to their teaching experiences (e. g. Bell, et al., 2010).

Second, this review limits its scope to empirical studies because one of the goals of this review is to provide some implications to policymakers. Empirical evidence might be more important when it is applied into making political decision for education system than just conceptual theories.

Third, the range of time period is limited from 2001 to 2013. That is, the present review only focuses on the studies that were conducted after the passing of the NCLB.

Based on these criteria, three electronic databases were searched between the years of 2001-2013: the Proquest Education Journal, ERIC (EBSCO) and JSTOR. Search parameters for the review were identified by initially focusing on a set of keywords to search the educational databases, including: elementary teacher knowledge and mathematics, elementary teacher content knowledge and mathematics and pedagogical content knowledge and mathematics. Studies that were presented as conference papers and dissertations were excluded for practical reasons; studies presented as conference papers may be incomplete and it is difficult to evaluate the quality of the researchers for dissertation studies.

2.2 Framework for synthesis

For synthesis that includes about 20 studies, it is better to describe each study explicitly and focus on the features that affect the implications than to draw conclusions by summarizing the studies' results (Wayne & Young, 2003). Therefore, I focused on specific details of each study and found a relationship among them.

The fundamental criterion for grouping studies is their research questions. The studies' research questions are related to each other to a greater or lesser detail. For example, in their research, Anderson and Kim (2003) asked, "What aspect of the knowledge based that prospective teachers develop in their teacher education programs makes them a successful mathematics teacher?" (p. 17). In contrast, Empson and Junk's (2004) research question is "What knowledge does teachers who have implemented a student-centered curriculum use to make sense of students' non-standard strategies?" (p. 122) Although the details are different, both research questions are basically asking how we can define teachers' knowledge for teaching mathematics.

Based on their relationship, I categorized them into four groups. The four groups are (a) how can we define elementary teachers' knowledge for teaching mathematics? (b) How can instruments for measuring elementary teachers' knowledge for teaching mathematics be developed? (c) What factors are to be related to elementary teachers' knowledge for teaching mathematics? And (d) what are international studies saying about elementary knowledge for teaching mathematics? For each group, description of all relevant studies and findings and interpretations are presented.

3. Findings

This section offers the findings from the present review. As explained in the previous section, studies were categorized into four groups based on their research question. For each group, description of all relevant studies and findings, interpretations, and implications are presented. I put forward results at a level of detail in order to balance content with consistency. That is the length of the explanation does not make judgments about the strengths and weaknesses of individual studies.

3.1 How can we define elementary teachers' knwoeldge for teaching mathematics?

3.1.1 Relevant studies and fiindings

For organizational purpose, this subsection includes three groups based on the main objectives from the research design of studies focusing on elementary teachers' knowledge for teching matheamtics related to the developing the instructional process (e.g., lesson development as well as lesson implementation), presenting mathematical concepts, and understanding about students' mathematical background.

3.1.1.1 Knowledge for teaching mathematics related to conducting instructional precess

Only two studies (Stylianides & Ball, 2008; Ball, Thames & Phelps, 2008) defined elementary teachers' knowledge for teaching matheamtics by analyzing teachers' daily tasks for teaching mathematics.

Stylianides and Ball (2008) analyzed 2 videotapes of elementary mathematics classrooms. The researchers found that the elementary teachers should be able to identify situations in which proof is called for, recognizing the important mathematical differences among these situations. In addition, the researchers asserted that elementary mathematics teachers should know how to provide appropriate opportunities for their students to engage in the class based on their understanding about the situations.

Ball, et al. (2008) documented and analyzed an entire year of mathematics teaching in a third grade public school classroom. From the analysis, the researchers found that teachers need knowledge for teaching mathematics when they organize methods or decide procedures before the class. In addition, knowledge for teaching mathematics is also required when they teach during the class and when they evaluate students' work or assignments after the class.

3.1.1.2 Knowledge for teaching matheamtics related to presenting mathematical concepts

Three studies (Turner, 2008; Izsa´k, 2006; Polly, 2011) defined elementary teachers' knowledge for teaching mathematics by focusing on the teachers' ways of presenting mathematics concepts.

Turner (2008) observed 12 beginning teachers' mathematics classrooms from their initial year to their third year in which the focus was on the teaching of algorithms. From the observation, the researcher discovered that differences did exist among teachers' knowledge for teaching mathematics of selecting representations. Elementary teachers who seemed to have a lack of knowledge were apt to choose representations that appealed to their eyes rather than that contributed in helping students understand mathematical concepts.

Izsa´k (2006) observed 2 sixth-grade elementary teachers' instruction. From the observation, Izsa´k suggested that teachers needed to assemble structures related to carrying out instruction with flexibility that are supported by drawn versions of the distributive property. Another important conclusion that Izsa´k made is that this kind of elementary teachers' knowledge for teaching mathematics is not adequate for understanding and appraising the diverse ways students present evidence of their learning of multi-level structures, when using drawings to reason about fraction multiplication situations.

Polly (2011) studied how teachers present mathematical concepts by using technology. From the observation of the 2 teachers' mathematics instruction, the researcher found that these teachers used technology only in ways that presented basic knowledge and did not facilitate higher thinking skills although they received preparation in how to use technology as a resource in the mathematics classroom. Consequently, the researcher suggested that teachers need special knowledge, which was referred to as *Technological Pedagogical and Content Knowledge* (TPACK) (p. 95).

3.1.1.3 Knowledge for teaching mathematics related with understanding about students' mathematical background.

Three studies (Empson & Junk, 2004; Anderson & Kim, 2003; Kleve, 2010) described elementary teachers' knowledge for teaching matheamtics related to students' mathematical background.

Empson and Junk (2004) interviewed 13 elementary teachers. From the analysis of data, the researchers showed that elementary teachers should have broad and deep knowledge of students' nonstandard strategies as well as have a way of deal with them. In addition, the researchers concluded that the teachers who have a deep comprehension of the children's way of understanding mathematics might teach mathematics better than those who do not.

Anderson and Kim (2003) defined elementary teachers' knowledge for teching mathematics as the ability to analyzed students' background knowledge and belief of mathematics from the literature review. The researchers argued

that teachers should have knowledge for teaching mathematics about analyzing students' background knowledge and beliefs about a mathematical topic. They concluded that elementary teachers' knowledge for teaching mathematics must be related to students' learning.

Kleve (2010) videotaped and analyzed a lesson about fraction taught by a 5th grade classroom teacher. By showing teacher's different explanations about the same topic, Kleve revealed that elementary teachers' knowledge for teaching mathemaetics related to teaching mathematics in various ways based on students' capabilities of understanding mathematics.

3.1.2 Interpretations and questions

A common perspective of the studies reviewed in this section suggests that elementary teachers' knowledge for teaching mathematics does not simply mean the understanding of mathematics content. In addition, the studies focused on elementary teachers' knowledge for teaching mathematics related to diverse aspects of instruction regarding planning and conducting mathematics lessons, using mathematical representations, and students' mathematical understanding. The sum of these studies emphasize that teachers should know how to express their knowledge for teaching mathematics based on students' level of mathematical understanding and background.

It is reasonable to assume that the findings from these empirical studies do contribute to our understanding of the importance of elementary teachers' knowledge for teaching mathematics. However, do these findings represent all aspects of elementary teachers' knowledge for teaching mathematics is a question that remains to be answered? As Fenstermacher and Richardson (2005) points out, instruction can be diversified because of resources, students and surroundings. However, the studies in this category have limits in which all aspects of mathematics instruction related to elementary teachers' knowledge knowledge for teaching mathematics is not covered. For example, showing proper examples, illustrations or manipulatives could be important for teaching mathematics, yet it is not the only way. Also, all of the studies focused on only one mathematics content area, number and algorithms. Diverse studies about the other areas show that there could be other elementary teachers' knowledge for teaching mathematics related to using materials. For instance, Polya (1985) highlighted the importance of mathematics teachers' questions when they teach problem solving to students. On the other hand, Clements (1998) emphasized elementary students' sensorial experience in order to form spatial sense.

In addition, all the research in this category did not include specific guidelines for students. However, various studies about characteristics of elementary mathematics students illustrate that it is hard to define stereotyping image of elementary mathematics students (e.g. Maloney, Risko, Ansari & Fugelsang, 2010; Gadanidis, Hughes & Cordy, 2011; Thornton, 1997). Although a teacher may use proper mathematics representations, it would be meaningless if students cannot understand them. In this case, another elementary teachers' knowledge for teaching mathematics might be needed that are different from knowledge defined above such as understanding students' previous mathematics knowledge.

3.1.3 Implications

The studies' findings in this section demonstrate what makes mathematics teachers different from mathematicians. Also, elementary teachers' knowledge for teaching mathematics about diverse mathematical representations related to students' grade level might help draw a distinction between elementary and secondary teachers. However, there are only a few empirical studies in this area, creating generalization issues. For instance, the studies presented in this section tended to focus on a particular grade level, conceptual area, or mathematical representation, making it difficult to generalize to a larger or similar teacher or student population at the elementary level. Therefore, more research is needed in this area.

Although there is still issue about generalization, the research of elementary teachers' for teaching mathematics is still meaningful. It might provide us some cues about what makes elementary mathematics teachers professionals. According the results of the research, even mathematicians who have high mathematical knowledge of contents may not teach well, if they do not have knowledge about how to teach mathematics to elementary school students. In addition, this might helps us to understand how elementary teachers' knowledge for teaching mathematics changed with flexibility according to students, surroundings, and recourses.

Furthermore, these findings might provide implications of how policymakers might develop pre-service and in-service elementary mathematics teacher education programs. Based on the research's finding the curriculum of the teacher education programs should be organized into three broad areas: how to plan or conduct mathematics lessons,

how to present mathematics concepts, and how to understand students' mathematical background.

3.2 How can instrument for measuring elementary teachers' knowledge for teaching mathematics be developed?

3.2.1 Relevant studies and fiindings

Only one research study (Hill, Schilling & Ball, 2004) focused on developing an instrument for measuring elementary teachers' knowledge for teaching mathematics. From the statistical analysis of the data that obtained from the survey with 138 items, Hill, Schilling and Ball (2004) concluded that elementary teachers' knowledge for teaching mathematics is measurable through paper-based tests. Based on the findings, this study suggested that elementary teachers' knowledge for teaching mathematics could be measured if every factors of the elementary teachers' knowledge for teaching mathematics is identified.

3.2.2 Interpretation and questions

In this category, only one study emerged from the database literature search. Therefore, it is hard to draw meaningful interpretations with other empirical studies. However, although there is one empirical study in this section, some questions arise from the findings. In the previous section, findings showed that even teachers who have high knowledge for teaching mathematics of contents may not teach well, if they do not know how to teach mathematics. Likewise, it is hard to say that to know the ways of teaching guarantee teaching well. Consider teachers who provide instruction perfectly based on what they know regardless of students' attentions or interests. Can we say that these teachers have high knowledge for teaching mathematics? Even worse, can an elementary mathematics teacher who learned teaching methods by rote teach mathematics well? Of course, teachers who have more knowledge for teaching mathematics have a higher chance to teach better than those who have no knowledge. However, as elementary teachers' knowledge for teaching mathematics was defined in instructional process, elementary teachers' knowledge for teaching mathematics also should be measured in the mathematics classroom.

3.2.3 Implication

The attempt to measure elementary teachers' knowledge for teaching mathematics might have a huge implication for policymakers. As Hanushek (2002) suggested that present teacher certification couldn't ensure high-quality teachers, policymakers should find ways of reforming the certification system. Developing appropriate instruments to measure elementary teachers' knowledge for teaching mathematics might propose solutions to this issue, if reliable measures are used. Also, the results of this study, which indicates elementary teachers' knowledge for teaching mathematics are measurable by paper-based tests, could provide some clues to policymakers. However, more extensive studies are needed in this area to develop measures for elementary teachers' knowledge for teaching mathematics that covers a wide range of mathematics areas. Therefore, policymakers should invest in developing methods of measuring elementary teachers' knowledge for teaching mathematics. Also, policymakers should find ways to relate those measures to teacher certification tests.

As presented above, more studies are need in this area. Not only research on testing possibility of applying pencil-and-paper tests to other mathematical areas, but also research of developing various methods of measuring elementary teachers' knowledge for teaching mathematics is needed. As Flanders (1961) one of the first developers of effectively analysis tools for measuring teaching situations based on interaction between a teacher and students, various approaches including paper-based tests are needed to develop better instruments that measure elementary teachers' knowledge for teaching mathematics. However, measurement should not be in any way interpreted as an absolute parameter of representing elementary teachers' knowledge for teaching mathematics. Again, it could provide some criteria that might help us understand elementary teachers' knowledge for teaching mathematics in part, but may not represent the whole of their knowledge.

3.3 What factors are to be related to elementary teachers' knowledge for teaching matheamtics?

3.3.1 Relevant studies and fiindings

For structural purpose, this section includes three subgroups focusing on what factors are affected by elementary

teachers' knowledge for teaching mathematics, what factors affect on elementary teachers' knowledge for teaching mathematics, and what aspects of elementary school correlate with elementary teachers' knowledge for teaching mathematics.

3.3.1.1 What factors are affected by elementary teachers' knowledge for teaching mathematics?

Three studies (Hill, Rowan & Ball, 2005; Tanase, 2011; Hill, 2008) focused on the effect of elementary teachers' knowledge for teaching mathematics.

Hill, et al. (2005) collected data about students from their assessments and data on teachers through the teacher self-report instrument. From the analysis of data, the researchers found that elementary teachers' knowledge for teaching mathematics for teaching had a positive effect on student mathematics achievement. However, it was also found that teachers' knowledge for teaching mathematics had little effect on minority students' academic achievement.

Tanase (2011) observed 4 first grade teachers' lessons about teaching place value concepts. The researcher interviewed all teachers in order to evaluate their knowledge for teaching mathematics and gave students paper-and-pencil tests at the end of the semester. After comparing the results from measuring both teachers' knowledge and student achievement, the researcher found that there is positive correlation between teachers' knowledge for teaching mathematics and student achievement. However, the researcher found that there is an exception. Students who already received low level from the previous achievement tests still performed at or below average levels in their final test regardless teachers' knowledge for teaching mathematics.

Hill (2008) chose 10 teachers who taught various grades from second to sixth. The researcher measured these teachers' knowledge for teaching mathematics with paper-based tests. In addition, the researcher evaluated the quality of their mathematics instruction by using self-developed rubrics. After that, the researcher calculated correlation between scores of teachers' knowledge for teaching mathematics and scores of the quality of instruction statistically. As a result, the researcher found that there is a positive correlation between elementary teacher's knowledge for teaching mathematics and the quality of instruction.

3.3.1.2 What factors affect elementary teachers' knowledgefor teaching mathematics?

Three studies (Hill, 2010; Margolinas, Coulange & Bessot, 2005; Bell, Wilson, Higgins & McCoach, 2010) emphasized what factors affect elementary teachers' knowledge for teaching mathematics.

Using a sample of teachers (n = 625) from a national database, Hill (2010) investigated the relationship between elementary teachers' knowledge for teaching mathematics and their own educational background. In order to evaluate elementary teachers' knowledge, Hill developed an instrument that is referred to *mathematical knowledge for teaching measures* (MKT measures) (p. 545) based on the researcher's previous research. Also, Hill carried a series of questions in order to estimate teachers' experiences (e. g. years of experience, professional development experience). Hill combined teachers' MKT scores and their experience and calculated correlation between them statistically. Consequently, Hill saw that the analysis did not illustrate a significant relationship between MKT and mathematics-related professional development experiences.

A study preceding Hill (2010) was conducted by Margolinas, et al. (2005). The researchers selected 2 previous studies purposely and used their data. From the analysis of the data, the researchers found that teachers kept modifying and developing their knowledge for teaching mathematics while they were observing their own students' replies and activities during the class. Also, the researchers found teachers improved their knowledge for teaching mathematics related students' ways of dealing with mathematical problems based on their teaching experience.

On the other hand, Bell, Wilson, Higgins and McCoach (2010) examined the increasing amounts of elementary teachers' knowledge for teaching mathematics through teacher education programs. The researchers investigated 111 elementary teachers' participation to the nationally disseminated professional development programs. Comparing test results between pretest and protest, results revealed that elementary teachers' knowledge for teaching mathematics improved through teacher education programs.

3.3.1.3 What aspects of elementary school correlate with elementary teachers' knowledge for teaching mathematics?

Only one study (Hill & Lubienski, 2007) concentrated on the aspects of elementary school related to elementary teachers' knowledge for teaching mathematics based on the analysis of results from an instrument administered to 533 teachers. The researchers calculated the teachers' scores regading their knowledge for teaching matheamtics with instrument and

compared them with the population of students of the school where the teachers worked. As a result, the researchers found that schools enrolling larger numbers of students who had low marks on their achievement test and Hispanic students tended to employ teachers who had slightly less knowledge for teaching mathematics than their counterparts on average.

3.3.2 Interpretation and questions

A common viewpoint of the studies reviewed in this section is that there exist elementary teachers' knowledge for teaching mathematics, and it is related to diverse aspects of elementary education. In particular, elementary teachers' knowledge for teaching mathematics is connected to student achievement, including teacher certification and teacher education programs, teacher experiences, and student demographics in an elementary school.

Regarding student mathematics achievement, three studies (Hill, et al., 2005; Tanase, 2011; Hill, 2008) suggested that elementary teachers' knowledge for teaching mathematics have an affect on student achievement. However, there are limitations regarding the effect of elementary teachers' knowledge for teaching mathematics. While elementary teachers' knowledge for teaching mathematics did affect most student mathematics achievement, there was little effect of elementary teachers' knowledge for teaching mathematics on minority students and students that received low marks from previous achievement tests.

Although this review considered only three studies, which focused on the affect of elementary teachers' knowledge for teaching mathematics, the findings from these studies showed different views. Concerning the affect of teacher experience, two studies (Hill, 2010; Magolinas, Coulange & Bessot, 2005) reached the same conclusion that there is a positive correlation between teacher experience and elementary teachers' knowledge for teaching mathematics. However, the conclusions from these two studies (Hill, 2010; Higgins & McCoach, 2010) differed from each other regarding whether there is an affect of teacher education programs on improving elementary teachers' knowledge for teaching mathematics or not.

Only Hill and Lubienski's (2007) study concentrated on the relationship between elementary teachers' knowledge for teaching mathematics and student demographics. Their research found that teachers in schools with higher proportions of students who are Hispanic and received low mathematics achievement scores had a lack of knowledge for teaching mathematics.

The findings in this section generated some controversy. Hill's (2008) results, suggesting that there were little affect on teacher education programs or existence of certification on teachers' knowledge for teaching mathematics, threatens the reason for maintaining teacher education programs or teacher certification systems. Of course, there is a different opinion about the affect on teacher education programs. However, the most important thing is if the affect on teacher education programs or teacher certification is not completely reliable. A question that needs to be answered then is how do elementary schools employ high-qualified teachers with strong knowledge for teaching mathematics?

On the other hand, the other studies revealed that teaching experience do affect teachers' knowledge for teaching mathematics, lead to a question about how beginning elementary mathematics teachers may improve their knowledge for teaching mathematics. Also, based on results from the other studies in this section, elementary teachers' knowledge for teaching mathematics impacts student achievement. Subsequently, students who learn mathematics from a teacher who is inexperienced might receive lower scores regardless of their ability than students who learned from more experienced teachers. This issue should be discussed because the way to handle this issue might be important in terms of school equality.

The other question regarding the findings in this section is about the distribution of teachers. If elementary teachers' knowledge for teaching mathematics is not helpful in improving mathematics achievement of minority students or students who had low marks on their pre-test, what are the problems for those students learning from teachers who have poor knowledge for teaching mathematics? This issue also leads to doubts about teachers' knowledge for teaching mathematics. If elementary teachers' knowledge for teaching mathematics is only applied to students who already have middle or high mathematics achievement or do not belong to a minority group, there is unquestionably a missing part on elementary teachers' knowledge for teaching mathematics.

3.3.3 Implication

In order to handle these issues, various and numerous research is needed. In order to clarify the effect of teacher education programs and teacher certification in terms of elementary teachers' knowledge for teaching mathematics, extensive research should be conducted across national teacher education programs and teacher certification. Also,

additional research is needed about elementary teachers' knowledge for teaching mathematics in order to find the way to improve minority and low academic mathematics achievement.

In addition, more research including diverse approaches on the affect of teachers' knowledge for teaching mathematics is needed. For example, Ma (1999) recommended that elementary teachers could learn from their fellow teachers. Richardson (2000) suggested that teachers' field research might lead to improved teachers' knowledge about their field. If studies could find various ways to develop elementary teachers' knowledge for teaching mathematics, it might be applied to teacher education programs

The findings from the studies in this section are also meaningful to policymakers. Policymakers should find the way to retain reliability of teacher education programs and teacher certification related to elementary teachers' knowledge for teaching mathematics. Too, the results that elementary teachers' knowledge for teaching mathematics may improve through their experience could provide a chance to think about the tenure system in elementary schools. If teachers could improve their knowledge based on their experiences, there is a possibility that more experienced teachers would teach better. In addition, for the same reason, pre-service teacher education programs should provide more filed experiences for prospective teachers in classrooms with teachers who model strong knowledge for teaching mathematics.

3.4 What are the international studies saying about elementary teachers' knowledge for teaching matheamtics?

3.4.1 Relevant studies and fiindings

This section presents studies that were conducted in other countries, or comparative research between U.S and non-U. S. Countries. Four studies (Li & Huang, 2008, Ng, 2011; Dalaney, Ball, Hill, Schiling & Zopf, 2008; Cai, 2005) presented in this section were conducted in other countries.

Li and Huang (2008) used a paper and pencil test to detect if these teachers' knowledge for teaching mathematics related to their understanding of mathematics representations in the classroom. In this study, the 18 Chinese teachers had to choose the right answer to mathematics questions or write about how to explain the solution to students. The researcher identified teachers' explanations and analyzed elementary knowledge for teaching mathematics related to both mathematics contents and teaching mathematics. After comparing these results with teachers' experience, the researcher concluded that the more experienced elementary teachers had higher scores in knowledge for teaching mathematics.

Ng (2011) also found that there was significant difference between groups of teachers based on experience in teaching. The researcher compared 184 Indonesia elementary teachers' records about teaching experience, including grade level with their knowledge for teaching mathematics. The researcher used a paper based measurement instrument, which were developed by LMT Project in United States. The researcher found that teachers who had taught diverse grades received a higher score on their tests.

Dalaney, et al. (2008) examined 4 Ireland elementary teachers' knowledge for teaching mathematics with translated measurement instrument, including multiple-choice items, which were provided by LMT Project. The researchers found some intersection between Ireland and U.S. elementary teachers' knowledge for teaching mathematics. However, the translated measurement instrument was not constantly applicable to measure Ireland elementary teachers' knowledge for teaching mathematics. The researchers found the reason for this inconsistency might be cultural differences of mathematics education (e.g. differences in mathematical language).

Cai (2005) compared elementary teachers' knowledge for teaching mathematics between U.S and China in a different way. In this research, Cai analyzed 11 U.S. and 9 Chinese teachers' lesson plans and interviewed all teachers based on the use of representations for mathematics concepts. From the analysis, the researchers discovered that Chinese teachers used representations to explain the process of algorithm than concepts and U.S teachers was the reverse. Cai found causes in differences regarding cultural values of mathematics instruction.

3.4.2 Interpretation and questions

Although there are conflicting opinions about that whether mathematics is isolated from the culture, the teaching and learning of mathematics is certainly a cultural activity (Stigler & Hiebert, 1999), international research about elementary teachers' knowledge for teaching mathematics has a similar tendency with studies that targeted U.S elementary teachers. International studies also assumed that elementary teachers' knowledge for teaching mathematics is measurable, and that there is a positive correlation between elementary teachers' knowledge for teaching mathematics and teacher experiences.

There are two studies that attempt to measure teachers' knowledge for teaching mathematics by using measurement instruments that were developed in the United States. Ng (2011) used tests developed by LMT Project in U.S. without any difficulty, while Dalaney et.al (2011) pointed out differences in teaching mathematics between two countries. In line with Dalaney et.al opinion, Cai (2005) also found that there are differences between what is considered valuable in mathematics education based on the culture.

The attempt to measure international elementary teachers' knowledge for teaching mathematics and compare the results to U.S. elementary teachers' knowledge for teaching mathematics might be meaningful for international comparison of teacher quality in terms of elementary teachers' knowledge for teaching mathematics. However, the question that still remains is how to develop measurement instruments that overcome cultural differences. As Hill (2008) pointed out, elementary teacher knowledge for teaching mathematics is related to their belief about mathematics. If measurement instruments ask about mathematics instruction containing a value judgment about mathematics education, it will be difficult to remain relevant.

3.4.3 Implication

Although there might be validity issues in developing international instruments for measuring international elementary teachers' knowledge for teaching mathematics, the attempt to compare international elementary teachers' knowledge for teaching mathematics is still meaningful. Recently, international tests for comparing student mathematics achievement became more important, and have provided implications about mathematics education to countries participating in the assessment (Grouws & Cebulla, 2000). Similarly, if we could measure international elementary teachers' knowledge for teaching mathematics, we might obtain more relevant information about mathematics education. Also, if we make a comparison between international elementary teachers' knowledge for teaching mathematics and international student mathematics achievement, we might obtain more pertinent information about the relationship between teachers' knowledge for teaching mathematics and student achievement.

Therefore, more research is needed in this area to develop international measurement instruments that can overcome cultural differences. In addition, policymakers from each country should work together in order to develop reliable measurement instruments. However it should not be interpreted as a standard in which to rate nations' elementary mathematics teachers' quality. Because there are culture differences among countries in the way mathematics is taught and the value of mathematics education, at this time elementary teachers' knowledge for teaching mathematics should not be representative of teacher quality.

4. Conclusion and Implications

Recent studies about elementary teachers' knowledge for teaching mathematics could contribute to how to improve elementary teachers' professionalism. Elementary teachers might teach mathematics better than mathematicians because elementary teachers know how to teach mathematics to elementary students. Also, the studies revealed that if we could identify elementary teachers' knowledge for teaching mathematics related to teaching, then we might find ways of improving elementary mathematics teacher quality that is connected to teachers' knowledge for teaching mathematics. However, there are not many empirical studies in this filed. Therefore, more research about elementary teachers' knowledge for teaching mathematics is needed.

In addition, the number of scholars conducting research in the area of elementary teachers' knowledge for teaching mathematics is few. In completing the literature review search the same scholars for different studies seemed to emerge. Therefore, not only do more research in this area needs to be done, but more important, there needs to be an increase in the number and type of scholars doing this work. Expanding the field of researchers will provide new ways of examining elementary teachers' knowledge for teaching mathematics and at the same time may provide critical information about affects and factors of knowledge for teaching mathematics.

The research about elementary teachers' mathematics knowledge may offer suggestions to policymakers. The research's results that suggest elementary teachers knowledge for teaching mathematics might affect students' achievement, it might be measured by tests, and certain programs might improve it might be meaningful to policymakers. These results could offer suggestions to policymakers about how they might improve pre-service or in-service teachers education programs. Also, policymakers could find some implications from the findings to improve teacher certification.

However, it is highly likely that elementary teachers' knowledge is misleading in terms of student mathematics achievement. Results of the studies reviewed here propose that elementary teachers' knowledge for teaching mathematics fit the necessary conditions for improving student achievement in mathematics, but do not provide the

sufficient condition for it. For example, the finding from Hill's (2010) research shows that elementary teachers' knowledge for teaching mathematics has little effect on students with low marks. Also, the explanation about the effectiveness of elementary teachers' knowledge for teaching mathematics is not sufficient.

Since elementary mathematics education was defined as one of scholarship, countless mathematics educators have produced various theories and methods to improve elementary students' mathematical understanding. However, opinions still differ about the way to provide effective teaching of mathematics. The reason is that the objectives of mathematics education are human beings who are infinitely complex. Likewise, elementary teachers' knowledge for teaching mathematics is very difficult to defined in a single sentence. Therefore, policymakers need wisdom in order to provide absolute meaning of what is elementary teachers' knowledge. However, at the same time, policymakers should keep investing research and teacher education programs in order to improve elementary teachers' knowledge for teaching mathematics.

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