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The Nature of Misconceptions and Cognitive Obstacles Faced by Secondary School Mathematics Students in Understanding Probability: A Case Study of Selected Polokwane Secondary Schools

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Abstract

The study investigated the probabilistic misconceptions of South African students. A questionnaire was administered to a group of 74 students from grades 10, 11 and 12 selected randomly from 5 schools around Polokwane and analysed using SPSS version 22. Five groups of misconceptions were identified. The Structure of the Observed Learning Outcome (SOLO) taxonomy was used in this study to describe students' hierarchical understanding levels on the concept of probability. It was found that, generally there was no significant improvement in developmental level from grades 10 to 12. Overally, the mean of correct responses on the test problems was 2, 1411. The modal cognitive level on the individual test items was 2 which indicate that participants had some evidence of the use of probability principles and appropriate quantitative information is evident, but they may be incomplete or are incorrectly used. All participants had the basic understanding of the concept of probability and could carry out simple probability calculations. Participants of all levels showed evidence of the equiprobability bias (miscounting of outcomes in questions concerning theoretical probability), exhibited ignorance of the effect of sample size and were seldom successful on counter intuitive conditional probability problems. Gender differences were observed. The overall correct response rate of males (56.3%) was significantly greater than that of females (51.6%). Males and females also tended to answer differently, based on the type of question; many of these differences were statistically significant.

Keywords: *Probability, cognitive obstacles, misconceptions, mathematics.*

1. Introduction and Background of the study

The difficulties experienced by pupils with probabilistic concepts have been confirmed by a number of recent studies. Probability is young area of mathematics in the South African secondary school curriculum. There is a growing trend in South Africa and world over to include probability in the school curriculum. Probability is part of the new Curriculum Assessment Policy Statements (CAPS) curriculum in South Africa (Department of Education, 2012). In the past it was regarded as an enrichment topic targeted for more "able" pupils who often encounter related questions in mathematics competitions. Some elements of probability were included in Curriculum 2005, but the emphasis was on the knowledge of ways of counting and understanding of probability of concepts. Literature advocates for the need to include probability but not much is known about how pupils struggle to grasp the topic and about the most effective pedagogical approaches.

Probability is the study of random events. It is used in analysing games of chance, genetics, weather prediction, and a myriad of other everyday events. A number of researchers Gal (2005) and Jones (2005) highlight the reasons for including probability in the secondary school mathematics curriculum. These reasons are related to the usefulness of probability for daily life, its instrumental role in other disciplines, the need for a basic stochastic knowledge in many professions, and the important role of probability reasoning in decision making. Students will meet randomness not only in the mathematics classroom, but also in biological, economic, meteorological, political and social activities (games and sports) settings. The understanding of probabilistic concepts does not appear to be easy, given the diversity of representations associated with this concept. Probability is difficult to understand for various reasons, including disparity between intuition and conceptual development (Chadjipadelis & Gastaris, 1995). According to Batanero, Biehler, Carmen, Maxara, Engel and Vogel (2005) probability is increasingly taking part in the school mathematics curriculum; yet most teachers have little experience with probability and share with their students a variety of probabilistic misconceptions.

Today's information-rich society need to have an understanding of probability. People are faced more and more often with making decisions in an environment that involves uncertainty. Such an environment requires the understanding of probability as an important topic which has real-life applications. Despite the importance of probability, many children and adults hold misconceptions about probability (Jones & Tarr, 2007). Instruction in probability should provide experiences in which students are allowed to confront their misconceptions and develop understandings based on mathematical reasoning (Shaughnessy, 2003). However, teachers tend to omit topics from probability due to their own lack of experience or misconceptions (Jones & Tarr, 2007). The existence of student-held perceptions about probability may explain, in part, why learning probability is problematic. The problems associated with the learning of probability are not well-documented (Pratt, 2011; Gage, 2012).The spectrum of students' problems span from difficulty with proportional reasoning and interpreting verbal statements of problems, conflicts between the analysis of probability in the mathematics lesson and experience in real life, and premature exposure to highly abstract, formalised presentations in mathematics lessons(Sadaghiani, 2006).

Probability is a complex concept which has multiple dimensions. Probability can be interpreted descriptively using words such as never, impossible, unlikely, probably, certain, and so on, but how they are used in probability may be different from their everyday usage. There are three approaches to the concept of probability: theoretical, empirical and subjective. The literature concerning misconceptions about probability identified many misconceptions, such as representativeness, availability, outcome approach, equiprobability. Research has shown that most students do not understand the concepts that are involved in probability. Kaplan (2008) noted that students' failure to intuition about ideas of probability stems from their inability to handle rational number concepts and proportional reasoning which are used in calculations, reporting and interpreting probability concepts. Students' weaknesses in handling basic concepts involving fractions, decimals and percentages were also identified as a potential threat to understanding probability concepts (Tso, 2012). A research conducted by Hansen, McCann and Myers (1985) revealed that students' difficulties in translating verbal problem statements into algebraic sentences present difficulties as these do not rest on school mathematics.

Borovcnik (2012) noted that the abstract and formal nature of probability often cause students to develop a negative attitude towards probability concepts. Students' level of mental maturity and level of specific mathematics skills often hinder their understanding of probability concepts. A number of studies carried on the cognitive development of secondary school students indicate that most of them cannot operate on the formal operational level (Green, 1983; Stonewater & Stonewater, 1984). Green (1983) further noted that students' inadequate verbal ability often limits them from accurately describing probabilistic situations. Students need to develop skills in dealing with abstract concepts before being exposed to probabilistic reasoning.

Secondary school mathematics students often fall victim to misconceptions due to the conflict between their expectations based on mathematics and their intuitions rooted in experience. Cognitive biases are tendencies to think in certain ways. Cognitive biases can lead to systematic deviations from a standard way of doing things. Students develop concepts of probability without formally studying the discipline and some of their concepts are at variance with those taught in the classroom (Madsen, 1991).Good teaching entails replacing these informal conceptions with more normative ones. Probability naturally lends itself to plenty of fun, hands-on cooperative learning and group activities. Activities with spinners, dice, and coin tossing can be used to investigate chance events.

Teachers' lack of knowledge about probability was identified as a predictor of students' limited understanding of probability (Papaieronymou, 2009). Shaughnessy (2003) noted that the effectiveness of the probability instruction depends on those who teach it. However, most South African mathematics teachers have little experience about the topic (Mutodi & Ngirande, 2014). Much of the recent literature revealed that in South Africa many mathematics teachers lack a sound grounding in probability (Wessels & Nieuwoudt, 2011). This lack of grounding limits their confidence and competence in teaching data handling and probability.

The aim of this paper is to shed light on the nature of misconceptions, difficulties and obstacles exhibited by high school mathematics students. The identification of the difficulties displayed by students is desirable in order to organize in-service training programmes and workshops on preparing didactical situations which allow the students to overcome their cognitive obstacles. The analysis of students' cognitive obstacles should provide the teacher with a deeper understanding of pupils' reasoning. In order to provide effective instruction, mathematics teacher educators need to know the nature of such obstacles and prescribe effective intervention strategies

2. Problem Statement

Students in South African secondary schools have very limited knowledge and skills about probability (Bennie, 2005).

This is due to a number of factors including influences from both inside and outside the schools. The role of probability in decision making is still underestimated. Compared to other mathematical topics, probability was recently introduced in the syllabus to become an integral part of the mathematics curriculum. Given this background about students' probabilistic thinking, it is not surprising that students' knowledge about the topic is still very limited. Furthermore, previous studies came up with inconclusive results. Some indicated that there are very limited teaching resources, such as textbooks, activities and materials (dice, marbles…) provided by schools (Jun, 2000) and other results indicated teachers' lack of knowledge and little experience about probability was identified as a predictor of students' limited understanding of probability (Bernstein, McCarthy & Oliphant, 2013; and Papaieronymou, 2009). Students' limited experience with probability language compounds the problem as they often have difficulty in adequately explaining their thinking. Therefore it is necessary to sight see the nature of misconceptions and obstacles faced by secondary school students in solving probability problems.

3. Objectives of the study

The objectives of this study were:

i. to explore the nature of misconceptions and obstacles faced by secondary school students in an attempt to solve probability problems;
ii. to determine the students' cognitive level in relation to probability according to levels suggested by Watson and Collis (1993);
iii. to investigate the effect of some demographic variables on students' understanding of probability and,
iv. Give recommendations to the educators and policy makers on the best practices that can decrease student's difficulties in probability.

4. Research Questions

1. What kinds of misconceptions do secondary school mathematics students have regarding probability?
2. What is the nature of students' obstacles about probability and can some of them be identified as a springboard for the development of a desirable mathematical understanding?

5. Hypotheses

This study postulated that:

H^1: There is a significant difference between probability misconceptions and students' demographic variables (i.e. gender, grade level and home language).

6. Literature Review

6.1 Nature of students' misconceptions and obstacles

As Centeno (1988) points out, 'a difficulty is something that inhibits the student in accomplishing correctly or in understanding quickly a given item (p. 142). Difficulties may be due to several causes: related to the concept that is being learned, to the teaching method used by the teacher, to the student's previous knowledge, or to his ability'. A widely shared principle in educational psychology is stated by Ausubel, Novak and Hanesian (1983) that the most important factor that influences learning is the student's previous knowledge. Batanero, Godino, Vallecillos, Green and Holmes (2001) describe an obstacle as knowledge, not a lack of knowledge. S/he argued that students employ this knowledge to produce the correct answer in a given context, which they frequently meet. However when this knowledge is used outside this context, it generates mistakes. A contradiction between the students' thinking and the new idea is produced and inhibits the creation of knowledge. It is essential to identify the obstacle and to replace it in the new learning. However it can also be pointed out that even after the student has overcome the obstacle, it recurs from time to time due to lack of practice. Other difficulties experienced by students are due to a lack of the basic knowledge needed for a correct understanding of a given concept or procedure (Batanero, Godino, Vallecillos, Green & Holmes, 2001).

Fallacies in reasoning can occur because of violations in the application of laws of probability. Examples of such errors include stereotyping, confirmation bias, and matching bias. Many of these errors occur because of misconceptions about probability. Students often do not understand the laws of probability and form misconceptions through informal experiences outside the classroom (Hirsch &O'Donnell, 2001). Students may develop their own way of reasoning about uncertain events. Their lack of understanding may be due to a lack of experience with the mathematical laws of probability or because they use heuristics (Hirsch &O'Donnell, 2001; Kahneman, Slovic & Tversky, 1982). Although formal training and teaching are associated with improved reasoning (Hirsch & O'Donnell, 2001), many students who receive formal instruction continue to have misconceptions about the nature of probability and probabilistic reasoning. Misconceptions of probability are particularly resistant to elimination during typical classroom instruction as they appear to be of a psychological nature and are strongly held (Konold, 1991).

Shaughnessy (2003) claim that pupils do not approach the topic as blank slates, but have firmly established beliefs about chance long before they are taught any probability. Bennie & Newstead (2008) indicates, however, that these beliefs cannot be "checked in at the classroom door", that is, they conflict with 'school probability'. Therefore teachers must be aware of the intuitions that pupils might bring to the study of probability and replace these with "correct intuitions" in line with the "formal" approach to probability. However from a radical constructivist position, Konold (1991) stressed the need to reconcile the conflicts between the classroom and the outside world. There is still a lot of debate about different views of probability and different methodologies of teaching the topic.

7. Research Design and Methodology

7.1 Approach

This study is explorative and descriptive and uses a quantitative design to explore students' misconceptions, obstacles and difficulties related to probability. An explorative design is one in which the major emphasis is on gaining ideas and insights (Mutodi & Ngirande, 2014). Exploratory research is conducted to provide a better understanding of a situation and not designed to come up with final answers or decisions but to produce hypotheses and explanations about what is going on in a situation. A descriptive research design is one in which the major emphasis is on determining the frequency with which something occurs or the extent to which two variables covary. It provides a picture of the specific details of a situation, a social setting, a relationship (Neuman, 2011) or a picture of a phenomenon as it naturally occurs (Bickman & Rog, 2009).

7.2 Participants

Participants were both male and female grade 10, 11 and 12 students (n = 74) randomly selected from 5 secondary schools around Polokwane Province in South Africa. Efforts were made to include students from a wide range background to ensure the inclusion of students with varied level of exposure and experience with probability and in the degree to which they held misconceptions.

7.3 The Research Instrument

A set of 25 questions were selected and administered to the sampled respondents in order to examine the nature misconceptions. This set of questions formed the major research instrument of the study. Previous researches on misconceptions in probability utilised multiple choice formats (Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988; Fischbein, Nello, & Marino, 1991). However Shaughnessy (1992) argued that information from multiple choice items is often sketchy, incomplete and may not be ideal for clarifying students' thinking. Thus in the present research, all questions required open responses. The questions were administered to a sample of 74 students enrolled in grades 10, 11 and 12 mathematics CAPS Curriculum. The analysis of the responses in the present study identified four levels according to level of sophistication in a similar manner to that of Watson and Collis (1994).

Level 1. In interpreting probability situations no analysis or evidence of use of probability principles is demonstrated. Features may include: the use of irrelevant information, subjective judgements, disregarding quantitative information, guessing at random, belief in control of probability and absence of any reason. Responses that use recent experiences to predict or estimate probabilities, availability, are included in this level.

Level 2. Some evidence of the use of probability principles and appropriate quantitative information is evident, but they may be incomplete or are incorrectly used. Probabilistic reasoning based on the assumption of equal likelihood when none exists and the use of the representativeness heuristic is considered to be illustrative of this level.

Level 3. Probability principles are applied correctly used and an awareness of the role of quantification is evident. However, such quantification is precise or numerical.

Level 4. Probability principles are used correctly and relationships are explained quantitatively.

7.4 Reliability and Validity

Reliability refers to the degree of consistency of the data gathering instrument in measuring that which it is supposed to measure (Kimberlin & Winterstein, 2008). This degree of consistency is measured using Cronbach's alpha coefficient. Validity on the other hand, is a measure of internal consistency that shows the degree to which all the items in a test measure the same attribute (Masitsa, 2011). It is mandatory that assessors and researchers should estimate this quantity to add validity and accuracy to the interpretation of their data (Tavakol & Dennick, 2011). It ensures that each test item measures the same latent trait on the same scale. In this study, the Cronbach alpha was calculated for the 25-item questionnaire and found to be 0.677 which is viable since an acceptable value must lie between 0.70 and 0.90 (Hof, 2012).

To observe content validity, the questionnaire was constructed and structured so that the questions posed were clearly articulated and directed. All statements were formulated to eliminate the possibility of misinterpretations. This was followed by a pre-tested administered to 50 students who were excluded from the participants in the main study. The identified amendments were made to ensure the simplicity and clarity of some questions, making it fully understandable to the participants (Masitsa, 2011).

Table 1: Cronbach's alpha reliability coefficient

Cronbach's Coefficient Alpha		
Variable(s)	**Number of items**	**Alpha**
Probability Terms & Definitions	5	0.741
Theoretical Probability	5	0.827
Background(Venn Diagrams & Proportion)	5	0.763
Union And Intersection	5	0.605
Dependent & Independent Events	5	0.509
Overall questionnaire	**25**	**0.777**

8. Data Analysis

The test responses were marked according to the rubric and students' responses were recorded and coded according to Watson & Collis (1994)'s level of acceptability. A statistical computer package, SPSS version 22, was used to process the data. The techniques used during data analysis included descriptive statistics, t-tests and Analysis of Variance (ANOVA).

9. Results and Discussion

9.1 Descriptive statistics

Table 2: Demographic variables

Variable		Frequency	Percentages (%)
Gender	Male	30	59.5
	Female	44	40.5
Grade	10	24	32.4
	11	23	31.1
	12	27	36.5
Home language	Sepedi	50	67.6
	Shangane	14	18.9
	Venda	6	8.1
	Swati	4	5.4

Demographic data about the respondents shows that 30 (59.5%) were males and 44(40.5%) were females. The majority 27(36.5%) of the participants were grade 12 students. Sepedi dominated the home languages 50(67.6) while Shangane 14(18.9%) was also notable. The other languages were insignificantly represented.

The distribution of students' performance per question according to cognitive levels in table 3 below indicates that from the first category (Probability Terms & Definitions) most students were operating at level 1.Questions 2 and 5 were badly treated as only 5 and 7 students respectively operated at level 4. It can be noted that students struggled to identify the sample space after rolling a die as well finding the complement of a given probability. Students made a similar response in question 7 which also involves rolling a die. This incorrect response can be linked to lack of knowledge about the experiment together with its outcomes. A closer look at the distribution of respondents' cognitive levels frequencies in item 12 also revealed that 'sample space' poses a lot of difficulties.

Insufficient background on sets and set notation was also evident as respondents incorrectly interpreted the number of elements in a set [n(s)].Questions related to unions and intersections indicate that most participants operated at low cognitive levels as evidenced by high frequencies concentrated at the lower levels. Such a pattern can be observed from Questions 16 to 20. A similar trend was also evident for dependent and independent events. The overall mean and modal cognitive level were around 2 suggesting that some evidence of probability principles and appropriate quantitative information is evident, but they may be incomplete or are incorrectly used.

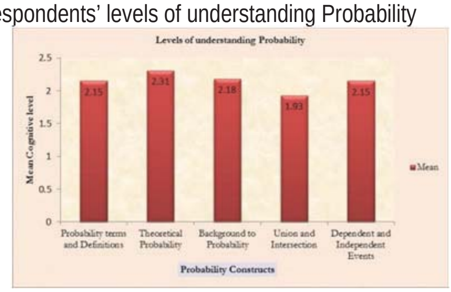
Table 3: Descriptive Statistics of respondents' levels of understanding Probability

Item	Description	Respondents' cognitive levels				Mean	Mode	Standard Deviation	Skewnes
		1	2	3	4				
PROBABILITY TERMS & DEFINITIONS									
Q1	Define the following terms: probability of an event, An experiment, An outcome, and sample space, An event	13	30	23	8	2.351	2.0	.8980	.170
Q2	Write down the sample space for rolling a single die numbered 1 to 6?	17	32	20	5	2.176	2.0	.8658	.299
Q3	Suppose that the probability of snow is 0.67, What is the probability that it will NOT snow?	21	22	21	10	2.270	2.0	1.0243	.215
Q4	What is the sample space for choosing a letter from the word probability?	25	26	16	7	2.068	2.0	.9698	.510
Q5	For any event A, P (A) + P (A') = _______, that is P (A') = ______- P (A).	36	21	10	7	1.838	1.0	.9935	.939
THEORETICAL PROBABILITY									
Q6	A bag contains 6 red, 3 blue, 2 green and 1 white balls. A ball is picked at random. Determine the probability that it is blue.	27	22	10	15	2.176	1.0	1.1391	.502
Q7	What is the probability of getting an even number when rolling a single 6-sided die?	25	34	13	2	1.892	2.0	0.7863	.542
Q8	What is the probability of landing on an odd number after spinning a spinner with 9 equal sectors numbered 1 through 9?	16	36	18	4	2.135	2.0	0.8163	.365
Q9	What is the probability of getting a 0 after rolling a single die numbered 1 to 6?	11	26	24	13	2.527	2.0	.9541	.018
Q10	A bag has 20 raffle tickets in it, numbered from 1 to 20. What is the probability of picking out an even number?	8	21	21	24	2.824	4.0	1.0117	-0.289
BACKGROUND(VENN DIAGRAMS & PROPORTION									
Q11	Let S denote the set of whole numbers from 1 to 16, X denote the set of even numbers from 1 to 16 and Y denote the set of prime numbers from 1 to 16. Draw a Venn diagram depicting S, X and Y.	13	23	18	20	2.608	2.0	1.0704	-0.048
Q12a	Pieces of paper labelled with the numbers 1 to 12 are placed in a box and the box is shaken. One piece of paper is taken out and then replaced. (a) What is the sample space, S? Find n(s).	33	25	9	7	1.865	1.0	.9697	.927
Q12b	Write down the set A, representing the event of taking a piece of paper labelled with a factor of 12? Find n (A).	30	18	10	16	2.162	1.0	1.1824	.494
Q12c	Write down the set B, representing the event of taking a piece of paper labelled with a prime number. Find n (B).	19	22	14	19	2.446	2.0	1.1365	.137
Q12d	Represent A, B and S by means of a Venn diagram.	33	29	6	6	1.797	1.0	.9063	1.098
UNION AND INTERSECTION									
Q16	Let ***E*** and ***F*** be events such that **Pr(E)=.6, Pr(F')=.3**, and **Pr (E∪F)=.8**. Find **Pr (E∩F)**	22	45	5	2	1.865	2.0	.7643	1.181
Q17	A jar has purple, blue and black sweets in it. The probability that a sweet chosen at random will be purple is 0.2 and the probability that it will be black is 0.6 If I choose a sweet at random, what is the probability that it will be purple or blue.	28	36	10	0	1.757	2.0	.6787	.342
Q18	If dice are the same colour, what is the probability of getting 2 or 3 on at least one of the dice?	27	31	15	1	1.878	2.0	.8268	.830
Q19	Suppose our experiment is flipping a coin three times in a row. Let B be the event that we do not get three heads in a row. Find P (B).	35	23	12	4	1.797	1.0	.9063	.872

Q20	Two fair dice are rolled. What is the probability that the sum of the values is a prime number?	30	22	19	3	2.297	1.0	1.1315	.143
	DEPENDENT & INDEPENDENT EVENTS								
Q21	A school decided that its uniform needed upgrading. The colours on offer were beige or blue or beige and blue. 40% of the school wanted beige, 55% wanted blue and 15%said a combination would be fine. Are the two events independent?	19	21	19	15	2.405	2.0	1.0844	.118
Q22	A jar contains 4 white marbles, 5 red marbles, and 6 black marbles. If a marble were selected at random, what is the probability that it is white? or black?	24	30	13	7	2.041	2.0	.9427	.624
Q23	If D and F are mutually exclusive events, with P(D) = 0,3 and P(D or F) = 0,94,find P(F).	29	19	11	15	2.162	1.0	1.1590	.488
Q24	Given Pr (E) = 0.5, Pr (F) =0.3, and Pr (E∩F) = 0.1. Determine if E and F are independent events?	18	24	15	17	2.419	2.0	1.0980	.180
Q25	A cloth bag has four coins, one R1 coin, three R2 coins and two R5 coin. What is the probability of first selecting a R1 coin and then selecting a R2 coin?	35	27	6	6	1.770	1.0	.9148	1.140

The respondents' cognitive level frequencies for each item were analysed and are shown in table 3 above. The results showed that the overall mean (M) cognitive level was (M) =2,1411, with a standard deviation (SD) of 0.3308 which according to the rubric indicates that some evidence of use of probability principles and appropriate quantitative information is evident, but they may be incomplete or are incorrectly used. The modal (Mo) cognitive level for each item was 2.0, acknowledging that most respondents agree that they experience some form of probability either in class or during individual study. These findings support a comment made by Spiegelhalter (2014) who echoed that probability is difficult to students because it is unintuitive and difficult. The first step in developing an understanding of probability is to acquire intuitions on how it works and relates to our natural sense of uncertainty. Results also indicate that students' lack of intuition on whether or not past experiences have an effect on the likelihood of future events.

Figure1: Descriptive Statistics of respondents' levels of understanding Probability



The results displayed in figure 1 above indicates most learners operate around level 2 which according to the rubric indicates some evidence of use of probability principles and appropriate quantitative information is present, but they may be incomplete or are incorrectly used. Probabilistic reasoning based on the assumption of equal likelihood when none exists and the use of the representativeness heuristic is considered to be illustrative of this level. Some attributes of level 1 were noted especially the use of irrelevant information, subjective judgements, disregarding quantitative information, guessing at random, belief in control of probability and absence of any reason. Most students operated at level 1 on aspects related to combining probabilities using intersections and unions.

9.2 Inferential Statistics

In order to test the contribution of the independent variables (gender, grade level and home language) against the dependent variable (cognitive level), two statistical models were used. These include the *t-test* for the difference between means and Analysis of Variance (ANOVA) for the analysis of the differences between group means.

9.2.1 Tests of Hypotheses

A *t-test* was conducted to compare if there is a significant difference between probability misconceptions and students' demographic variables (i.e. gender, grade level and home language). Results are shown in table 4 below. Results indicate that the mean score for male students' probability cognitive level was M_m= 2.2520 with (N_m =30, SD_m = 0.20934), which was slightly higher than the mean score of M_f=2.0655 with (N_f =44, SD_f = 0.37663) obtained from female students. It is apparent from table 4 that males fared better than their female counterparts.

Table 4: Gender &Probability Cognitive Level mean Differences

Probability Cognitive Levels			
Gender	**Mean**	**N**	**Standard Deviation**
Males	2.2520	30	0.20934
Females	2.0655	44	0.37663
Total	2.1411	74	0.33086

The following postulated hypotheses were envisaged to test if gender has a significant effect on probability cognitive levels:

H^0: There is no difference in probability cognitive levels between males and females.

H^1: There is a significant difference in probability cognitive levels between males and females.

A *t-test* was conducted to test whether there was a significant difference in probability cognitive levels between male and female students. The results for the test are shown in table 5 below (df=73, t = 24.457, p=0.00). Therefore, the null hypothesis was rejected since the p-value is less than 0.05. Hence we conclude that there is a significant difference in probability cognitive levels between males and females.

Table 5: t-Tests

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	95% Confidence Interval of the Difference
					Lower	Upper
Gender	24.457	73	.000	1.4054	1.291	1.520

To test if there are significant differences among students from different language backgrounds, an Analysis of Variance test was conducted to test the following hypothesis.

H^0 : There is no difference in probability cognitive levels among students from different language backgrounds.

H^1 : There is a significant difference in probability cognitive levels among students from different language backgrounds

Table 6: Anova-Home Language

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	65.678	29	2.265	.725	.818
Within Groups	137.457	44	3.124		
Total	203.135	73			

The results of the test in table 6 above show that (df = 29, df =44, F= .725, p=0.818).Therefore, we do not reject the null hypothesis since p>0.05 and conclude that there are no significant differences in cognitive levels among students from different language backgrounds. Results from the analysis indicated that demographic factors such as gender can predict probability cognitive levels of the students; however differences in grade and language backgrounds do not have significant effects on math anxiety levels. These observations seem to be consistent with findings from Kazima (2008) who observed that demographic factors gender were good predictors of students' probability cognitive levels.

Table 7: ANOVA-Grade

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	22.962	29	.792	1.248	.249
Within Groups	27.917	44	.634		
Total	50.878	73			

The results of the test in table 8 above show that (df = 29, df =44, F= .2.49, p=0.386).Therefore, we do not reject the null hypothesis since p>0.05 and conclude that there are no significant differences in cognitive levels among students from different grade cohorts. Results from the analysis indicated that demographic factors such as gender can predict probability cognitive levels of the students; however differences in grade and language backgrounds do not have significant effects on math anxiety levels. These observations seem to be consistent with findings from Kazima (2008) who observed that demographic factors gender were good predictors of students' probability cognitive levels.

10. Discussion of Results

It can be recalled that the purpose of this study was to explore the nature of misconceptions and cognitive obstacles faced by secondary school students in understanding probability and to examine the effect of demographic factors, which are gender, grade and home language. It was also hypothesized highly that there is a significant difference between probability misconceptions and students' demographic variables (i.e. grade level, gender and home language).

This study found that there was a significant difference in probability cognitive levels of students according to gender. Both descriptive statistics and inferential indicated that males had less misconceptions compared to their female counterparts. Findings in the present study are consistent with the findings of HodnikČadež and Maja (2011) who reported that differences in gender influenced probability tasks solving only to some extent. However, these findings contradict the findings of Jun (2000) which concluded that gender differences were insignificant. Long research history in this area has shown that male advantage in mathematics achievement is a universal phenomenon (Mullis, Martin, Beaton, Gonzalez, Gregory, Garden, & Murphy, 2000). In support to this, Mutodi and Ngirande (2014) also recognized that math interests of males are better than the females from secondary school onwards.

This study found that there was no significant difference in probability cognitive levels of students according to grade. Descriptive statistics indicated that students across the three grade levels experience the same difficulties. Further inferential statistics proved it significant. This is contrary to expectations, as grade 12 students were expected to perform better than lower grades students. A possible explanation to this might be that, teachers from the selected schools have difficulties in teaching the topic.

The study found that there were no significant differences in cognitive levels among students from different language backgrounds. The expectation was that since South Africa is a multi-cultural and multi-lingual country, students from different language backgrounds tent to experience difficulties differently. An important issue emerging from these findings is that other factors other than the identified ones have a direct influence on the development of students' misconceptions about probability.

11. Conclusions, Recommendations and Implications for Teaching

The findings of this study have significant implications for all stakeholders, including teachers, schools and curriculum planners. The fact that probability is also utilized in the areas close to everyday life of humans such meteorology, elections, actuarial science, etc, probability concepts and techniques need to be integrated in mathematics lessons as early as at the primary level, and not only in higher grades or even in high school, when the mindset of a student is already developed. This research confirms recommendations by Threlfall (2004) that relating everyday statements to probability language, answering probability or likelihood questions about a described situation, collecting and reflecting on empirical data may help to clear students' misconceptions about probability.

Most surveys emphasized students' lack of skills and background knowledge of mathematics as major contributors to the development of misconceptions. This study acknowledges that the concepts are difficult and require a high degree of sophistication which many students lack. Students lack sufficient background knowledge of topics such as ratio and proportion, which in turn present stumbling blocks in their understanding of probability. Helping students to develop the skill of self-regulation has been cited by Shaughnessy (1992) as a key to success in probability.

The instructional implication of this study is that teachers must determine where students' difficulties lie before they can intervene. If the concept is abstract and intrinsically difficult, the students will need more experience with it. If the students lack requisite mathematical skills, remedial work will be necessary first. The multiplicity of possible underlying reasons for students' difficulties greatly complicates the teacher's task. Yet proceeding without diagnosis almost surely will be fruitless. What is needed first is that teachers themselves be well informed. They should correctly understand the concepts and be aware of the different sources of difficulty that students may have.

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