Mathematics Teachers' Conceptions of Sense-Making in Word Problem Solving: A Case of Postgraduate Students at a University of Technology

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Abstract

In this paper, I discuss teachers' conceptions of what means to solve mathematical word problems that are connected to their daily lives. The role of social and cultural issues as well as participating teachers' were explored in relation to their understanding of what constitute modelling in word problem-solving. The results presented here are part of a wider study that followed a mixed-methods design with the qualitative results informing the quantitative data. For the purposes of this study, data that were collected via administering a test and conducting focus group with few selected teachers from a University of Technology in South Africa are presented. The main finding of this study is that teachers demonstrated a tendency to relegate multiple modelling techniques when they engaged with mathematical word problem solving. Moreover, the manner in which the participating teachers made meaning to word problem task seemed to be connected to issues reating to both the language of the solver as well as that used in the mathematical word problem task.

Keywords: *problem solving; sense-making; realistic considerations; word problems*

1. Introduction

Issues of teaching mathematics through understanding and use of problem solving as a pedagogical tool in mathematics classrooms seems to be a concern for many teachers in South African schools. The recently introduced Curriculum Assessment Policy Statement advocates for a mathematics lesson that is characterised by integration of problem solving accros the curriculum, with emphasis put on connecting classroom mathematics activities with out-of-school students' informal knowledge and experiences. However, little or no training of teachers in the suggested teaching methodologies took place. In fact, in few cases where curriculum training occurred, teachers were expected to understand the entire curriculum after a 3 to 4 days intensive training by untrained facilitators. It is against this background that the study reported in this article, sought to unpack teachers' multiple concepts of a notion of mathematical word problem solving, and how these problems could be modelled or taught in a 9th grade classroom.

The issues of sense-making in word problem solving were shown to have been connected to factors such as language of a problem solver, language used in the mathematical word problem statement, as well as the language used to teach and learn mathematics. Issues of language complexities are linked to the fact that South Africa is a contry with 11 official languages, yet English is predominantly used as an official language of learning and teaching in schools, a language which is not spoken at home (see Adler, 2001; Sepeng, 2011, 2013a; Setati, 2005). A major source of difficulty with mathematical word problems is the fact that the language used in the mathematics and English usage often differ in significant ways (Sepeng, 2013a). According to Kane (1968) there are four key difference between the two languages: (i) there are fewer redundancies in the language of mathematics; (ii) names given to mathematical objects usually have only a single denotation in mathematical language; (iii) adjectives are usually unimportant in mathematical language; and (iv) the grammar and syntax of mathematical language are far less flexible than is the case for general English. However, despite such differences being well-known, many children still encounter challenges in reading and writing mathematics because not much has been done by teachers to counter this (Durkin, 1978).

The South African National Curriculum Statement emphasises not only in the teaching of problem-solving, but pleads for (more) instructional attention to the acquisition of problem-posing skills (Department of Education, 2005). In developed countries, such as the US, there is also a growing interest among researchers in problem-posing (see e.g., Brown & Walter, 1993; Verschaffel, Van Dooren, Chen & Stessen, 2009). According to Verschaffel and his colleagues (2009), the most cited motivation for instructional and curricular interest in problem-posing is its perceived potential value in assisting students to become better problem solvers. To explore this potential value, several studies (Cai & Hwang, 2002; Ellerton, 1986; Silver & Cai, 1996) have been set up to investigate the relationship between word problems solving and word problem-posing. In these studies, students were given opportunities to generate a few problems starting from a situational description, a cartoon (or picture), and afterwards, the quality of the mathematical problems produced by the students was compared with their ability to solve problems. Other studies (English, 1997a, 1997b; Verschaffel, De Corte, Lowyck, Dhert, & Vandeput, 2000) have since revealed that having students engage in some activities related to problem-posing may have a positive influence not only on their word problem-posing abilities but also on their problem-solving skills and their attitude towards mathematical problem-solving and mathematics in general.

This topic of real-world knowledge and realistic considerations in students' solutions of arithmetic word problems has attracted the attention of many researchers in mathematics education. Several studies (Cai & Silver, 1995; Greer, 1993) have addressed this issue by looking at students' approaches to, and solutions of *non-standard* or *problematic* arithmetic word problems wherein the appropriate solution or mathematical model is neither obvious nor indisputable, at least if one seriously takes into account the realities of the context evoked by the problem statement.

An increasing number of researchers have consistently suggested that current school instruction given for arithmetic word problems is likely to develop in students' tendency to exclude real-world knowledge and realistic considerations from their solution processes (Cooper & Harries, 2005; Moreno & Mayer, 1999). For many children in elementary school, emphasis has been put on syntax and arithmetic rules rather than treating the problem statement as a description of some real-world situation to be modelled mathematically (Xin, 2009). For example, studies (Liu & Chen, 2003) conducted on 148 Chinese students from 4th and 6th grade, reported that only one fourth (26%) of the students' solutions of problems were from a realistic point of view (attending to realistic considerations). Almost half (48%) of the responses revealed a strong tendency to exclude real world knowledge, and in the rest of the cases, no answer was given. Word problem-solving in school contexts serves as a game under tacitly agreed rules of interpretation (Greer, 1997). According to Gatto (1992), these agreed rules are internalised in the students' minds through the socio-mathematical norm, or hidden curriculum of traditional schooling that could influence many aspects of the intellectual activities in schools.

2. Materials and methods

The main aim of the study discussed in this article was to explore issues of sense-making that play a role in grade 9 in-service teachers' mathematical problem-solving abilities. To address the above aim, a mixed method approach was used, viz. by gathering quantitative and qualitative data using test and focus group. These data were triangulated in order to make a stronger case in terms of the explanatory quality of this study (Tashakkori & Creswell, 2007), provides better argument (Creswell, 2008) and produce better understanding and verification (Creswell & Garrett, 2008; Creswell & Plano Clark, 2007). Mixed methods approaches intertwine both qualitative and quantitative methods in the same study (Lichtman, 2010) and combine elements of both qualitative and quantitative research approaches. According to Lichtman, these approaches are not parallel, but are an attempt to meld the best of quantitative and qualitative research designs. (Gilbert, 2006) suggests that a mixed methods approach intensifies the effect and enriches the adaptability of the research design.

2.1 Sample of the study

The main aim of the sampling in the study reported here was, among others, to select possible research participants because they possess characteristics, roles, opinions, knowledge, ideas or experiences that may be particularly relevant to this research (Gibson & Brown, 2009). The sample consisted of 40 grade 9 teachers from 40 secondary schools in 9 provinces of South Africa, four of which were urban schools and the rest of which were rural schools. The rural schools draw students from low economic status. All participants in the study were a convinience sample of teachers studying towards a Bachelor of Education (Honours) at a University of Technology in South Africa.

2.2 Procedure

The participants were given a word problem solving (PS) task in a form of a test that was administered in one of the lectures at a University of Technology. The PS task were coded using a schema that was an elaboration of the classification schema developed by Verschaffel et al. (1994) and satisfy the assessment standards as reflected in the South African Curriculum Assessment Policy Statement (CAPS). The questions are standardised, criterion-referenced measures of problem-solving skills with proven reliability and validity devised and verified (Maxwell, 1992). The classification schema comprised fourteen categories, the categories were reduced to three general categories: *realistic reaction (RR), no reaction (NR), and other reaction (OR).* These categories are explained and described later in the results section. The test items were as follow:

> *PS1: Two boys, Sibusiso and Vukile, are going to help Sonwabo rake leaves on his plot of land. The plot is 1200 square meters. Sibusiso rakes 700 square meters during four hours and Vukile does 500 square meters during two hours. They get 180 rands (R) for their work. How are the boys going to divide the money so that it is fair?*

Immediately after administering a test, a group of 8 participants were purposively selected to participate in a focus group. The discussion that took place during the focus group aimed at understanding the reasoning and thinking behind solving problems the way they did. The semi-structured questions of a focus group were asked as follow:

- How did you solve the problem?
- Why did you solve the problem the way you did? Explain.
- If the same thing happens to you, would you respond in the same way in real life as you did in solving the problem? Why?
- Your solution may not work in real life because of realistic considerations. Why did you answer that way?
- Please think about on what condition your answer could become realistic. Could you come up with any assumptions or explanations that can make your answers justifiable?
- Do you think the contexts used in the content of tasks (or problems) that you solved in the pre-test are familiar and/or relevant to your everyday life experiences? Why?

2.3 Data analysis

The process of data analysis involves making sense out of the data (Creswell, 2009); which requires the skill to depict the understanding of the data in writing (Henning, 2004). In other words, data in the study discussed in this article were analysed and interpreted via a process which involves preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data, representing the data, and making interpretation of the larger meaning of the data.

Students responses (or answers) to the problem-solving tasks (PS1 and PS2) were coded into three general categories: realistic reaction (RR), no reaction (NR), other reaction (OR), which were adapted from a schema developed by Verschaffel, et al. (1994). As noted earlier in chapter three, RR comprised all cases wherein a learner either gave the most accurate numerical response that also considered real-world aspects and context of the problem, or cases where there was an attempt to consider real-world situations without providing a numerically most correct response. On the other hand, OR were all those responses without real-world considerations, and situationally inaccurate responses with correct computations. NR were all the cases with no numerical responses and mathematically incorrect, without any further written responses that indicated that the learner was not aware of real-life aspects of the problem that made it impossible for him or her to solve the problem.

3. Results and discussion

Teachers responses to PS1 task were characterised by use of multiple models (see Extract 1) that demonstrated different interpretations and use of real-world knowledge and experiences in problem-solving. The following extract shows how students interpreted and solved this problem:

3.1 Extract 1

3.1.1 Teacher A:

> *Firstly I divided the Square metres that each an every one rakes by their total number of hours. I decided to use a mathematical method in order to get the correct solution and as sibusiso rake 700 square matres in 4 hours and Vukile rakes 500 square metres in 2 hours The number of square metres ÷ the total square metres × the amount that must be share , which give me 26 then × by the 4 hours which give us 104 for Sibusiso. Even for Vukile I did the same method but I just multiply it by 2 that give me R76*

3.1.2 Teacher B:

We compare the two quantities by using ratio. And I solve by using this way because sibusiso and vukile they did not doing the same square meters and they are not working equal hours sibusiso did rakes of 700 square meters for for hours and vukile did rakes of 500 meters for two hours.

3.1.3 Teacher C:

I divided the metres rake by each by a square metre of the yard which is 1200 square metre which will give the fair share of each boy where Sibusiso will get R105 as he rakes the largest side while Vukile gets R75

3.1.4 Teacher D:

I divided the plot which Sibusiso and Vukile did by the total square of the plot and multiplied by the money they will have to share

Extract 1 is used as an example of teachers' conceptions of what constitutes problem solving of mathematical word problems that requires more than mathematical computations. In most of the instances teachers responses to focus group discussions demonstrated a linear way of engaging with word problem-solving. The participating teachers suggested mainly a model of sharing the money that is influenced by *amount of work done* instead of considering other models that may include *time taken to do the work* and *payment by performance*. Mathematical terms used such as *compare the two quantities by using ratio* (Teacher B); *divided the metres rake by each by a square metre* (Teacher C), illustrated not only considering numerics when solving word problems but the nature of mathematical reasoning that seems to be connected to a problem-solvers' social and cultural background and influence. In other words, a notion of what constitutes a relevant context (Julie & Mbekwa, 2005; Sepeng, 2013b) in mathematical word problems included in the school mathematics curricula.

Table 1. Comparison of PS2 results per country at both teacher and learner levels

Models for sharing	No. of students (Sweden)	No. of students (South Africa)	No. of teachers (South Africa)
	n=78	*n=107*	*n=40*
A. Divide equally (R180/2)	33 (42%)	45 (42%)	0 (0%)
B. Amount of work done	27 (35%)	32 (30%)	29 (73%)
C. Time taken to do work	18 (23%)	11 (10%)	5 (13%)
D. Payment by performance	0 (0%)	7 (7%)	3 (7%)
E. Other	0 (0%)	12 (11%)	3 (7%)

There are almost similar patterns that emerge from the situation in which both Western students (e.g., Sweden and the US) and South African students have been challenged by the problematic word problems (Greer, 1997; Sepeng, 2011, 2013b; Verschaffel et al., 2009), the results in Table 2 showed that the participating teachers are not in the position to connect classroom mathematics activities to their own out-of-school informal knowledge about the mathematics during their problem solving processes. The data in Table 1 depict that majority (73%) of the teachers in this study only focused on one type of modelling technique viz. *amount of work done* and translated it to equating a "fair sharing" of the money. As such, only few (13%) and 7% of these teachers used *time taken to do work* and *payment by performance* models respectively, in their solution processes. The data in Table 1 appeared to suggest that particpating teachers did not include realistic considerations when solving PS1 task. In addition, teachers' problem-solving technigues were torn between two minds, one that considered machenical way of solving a mathematical problem that needs a straight forward computation, and those needing to consider both reality and contexts used in a mathematical task. It was evident that majority of them went for the former, which reflects pedagogies that are employed in their respective classrooms.

4. Conclusion

The study reported in this article seems to suggest that a recognition of the role of common sense reasoning and/or out-of-school knowledge in teaching mathematical word problems, coupled with considerations of the complementary roles of socio-cultural and multilingual nature of South African classrooms settings. In these classrooms, the pedagogies that are employed in the teaching and learning of mathematics do not only focus on the mathematics that is taught, but multiple issues that relate to how to empower themselves in order teach mathematics through problem solving. A key implication of the study in terms of teacher development in teaching mathematics with meaning and understanding via the use of contextual (or situational) problems is the explicit consideration of teachers' conceptions of contextual problems in multilingual classrooms. The findings in this article appear to suggest that if teacher development programmes are designed and structured in a way that empower mathematics teachers with knowledge and skills that promote understanding of the contextual role of problem situation in (word) problem-solving, they would be better placed to bridge the gap between school and everyday mathematics, as well as gaps between, home, school and mathematical language, whether using mother tongue or English.

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