Maximization of Profit in Manufacturing Industries Using Linear Programming Techniques: Geepee Nigeria Limited

Fagoyinbo, I. S. *
Akinbo, R. Y. *
Ajibode, I. A.*

* Department of Mathematics and Statistics, Federal Polytechnic, Ilaro,Ogun State, Nigeria

** Olaniran, Y.O.A

** Department of Marketing,Federal Polytechnic, Ilaro,Ogun State, Nigeria

Abstract *Any organization set up aims at maximization of profit from its investment from minimum cost of objective function. This research work applies the concept of revised simplex method; an aspect of linear programming to solving industrial problems with the aim of maximizing profit. The industry GEEPEE Nigeria Limited specialises in production of tanks of various types. Four different types of tank were sampled for study these are the Combo, Atlas, Rambo and Jumbo tanks of various sizes. Based on the analysis of the data collected it was observed that, given the amount of materials available Polyethylene (Rubber) and Oxy—acetylene (Gas) used in the production of the different sizes of the product, Combo tanks assures more objective value contribution and gives maximum profit at a given level of production capacity.*

Introduction

Linear Programming is a subset of Mathematical Programming that is concerned with efficient allocation of limited resources to known activities with the objective of meeting a desired goal of maximization of profit or minimization of cost. In Statistics and Mathematics, Linear Programming (LP) is a technique for optimization of linear objective function, subject to linear equality and linear in equality constraint. Informally, linear programming determines the way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model and given some list of requirement as linear equation. Although there is a tendency to think that linear programming which is a subset of operations research has a recent development, but there is really nothing new about the idea of maximization of profit in any organization setting i.e. in a production company or manufacturing company. For centuries, highly skilled artisans have striven to formulate models that can assist manufacturing and production companies in maximizing their profit, that is why linear programming among other models in operations research has determine the way to achieve the best outcome (i.e. maximization of profit) in a given mathematical model and given some list of requirement represented as linear equations.

Linear programming can be applied to various fields of study. Most extensively, it is used in business and economic situation, but can also be utilized in some engineering problems. Some industries that use linear programming models include transportation, energy, telecommunications and production or manufacturing companies. To this extent, linear programming has proved useful in modelling diverse types of problems in planning, routing, scheduling assignment and design. David (1982), Nearing and Tucker (1993) noted operational research is a mathematical method developed to solve problems related to tactical and strategic operations. Its origins show its application in the decision-making process of business analysis, mainly regarding the best use for short funds. This shortage of funds is a characteristic of hyper-competitive environments. Although the practical application of a mathematical model is wide and complex, it will provide a set of results that enable the elimination of a part of the subjectivism that exists in the decision-making process as to the choice of action alternatives (Bierman and Bonini, 1973).

Linear Programming deals with special mathematical problems by developing rules and relationships that aim at the distribution of limited funds under the restrictions imposed by either technological or practical aspects when an attribution decision has to be made (Andrade, 1990). Generally, Linear Program can be written in a canonical form for profit maximization as:

Max (Z) = C^T X

Subject to:

AX $\leq$ b

From the model above, x represent the vector of variables (to be determined) while c and b are vectors of known matrix of coefficient. The expression to be maximized is called the objective function (C^T in this case). The equation AX $\leq$ b is the constraint which specifies a convex polytope over which the objective function is to be optimised.

Linear Programming is a considerable field of optimization for several reasons. Many practical problems in operations problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming such as network flow problems and multi commodity flow problems are considered important enough to have generated much research on specialised algorithm for their solution. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory such as Duality, Decomposition and the importance of convexity and its generalizations. Standard form is the usual and most intuitive form of describing a linear programming problem. When the problem involves “n” decision-making variables and “m” restrictions, the model can be represented mathematically in the form of either maximization or minimization of the object function (Corrar and Teophilo, 2003). For instance, for a maximization problem: it consists of the following three namely: A linear function to be maximized:

Max(Z) = $C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_nX_n$

Problem constraints of the form:

$A_{11}x_1 + A_{12}x_2 + A_{13}x_3 + \ldots + A_{1n}x_n$ (≤ or≥) b_1

$A_{21}x_1 + A_{22}x_2 + A_{23}x_3 + \ldots + A_{2n}x_n$ (≤ or≥) b_2

$A_{m1}x_1 + A_{m2}x_2 + A_{m3}x_3 + \ldots + A_{mn}x_n$ (≤ or≥) b_m

Non negativity variables:

$x_1, x_2, x_3, x_4 \geq 0$

Purpose of the Study

The main purpose of this study is to critically examine at least four of the products produced in GeePee Nigeria Limited. To effectively estimate which of these products must be given more attention or produced more in other to maximize profit. In the course of this study, linear programming technique will be used to make the best possible use of the total available productive resources of GeePee Nigeria Limited (such as time, material, labours) etc. In the same vein, in a production industry like GeePee Nigeria Limited bottlenecks may occur. For example in the factory, a product may be in great demand while others may lie idle. As such, this research work is aimed at highlightening such bottlenecks i.e. identifying the product that must be produced more in other to maximize profit in GeePee Nigeria Limited.

Moreso, this research work will assist the management of GeePee Nigeria Limited to make valid decision with the technique of linear programming used in this research work so as to make an objective decision for reduction in wastage of resources like time, money, materials etc. may be avoided.

Literature Review

Linear Programming was developed as a discipline in the 1940's, motivated initially by the need to solve complex planning problems in war time operations. Its development accelerated rapidly in the post war periods as many industries found its valuable uses for linear programming. The founders of the subject are generally regarded as George B. Dantzig, who devised the simplex method in 1947, and John Von Neumann, who establish the theory of duality that same year. The noble price in economics was awarded in 1975 to the mathematician Leonid Kantorovich (USSR) and the economist Tjalling Koopmas (USA) for their contribution to the theory of optimal allocation of resources, in which linear programming played a key role. Many industries use linear programming as a standard tool, e.g. to allocate a finite set of resources in an optimal way. Example of important application areas include Airline crew scheduling, shipping or telecommunication networks, oil refining and blending, stock and bond portfolio selection.

The problem of solving a system of linear inequality also dates back as far as Fourier Joseph (1768 – 1830) who was a Mathematician, Physicist and Historian, after which the method of Fourier – Motzkin elimination is named. Linear programming arose a mathematical model developed during the Second World War to plan expenditure and returns in other to reduce cost to the army and increase losses to the enemy. It was kept secret for years until 1947 whn many industries found its use in their daily planning. The linear programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979 but a large theo vertical and practical breakthrough in the field came in 1984 when Narendra Karmarkar (1957 – 2006) introduced a new interior point method for solving linear programming problems. A lot of applications was developed in Linear programming these includes: Lagrange in 1762 solves tractable optimization problems with simple equality constraint. In 1820, Gauss solved linear system of equations by what is now called Gaussian elimination method and in 1866, Whelhelm Jordan refined the method to finding least squared error as a measure of goodness-of-fit. Now it is referred to as Gausss-Jordan method. Linear programming has proven to be an extremely powerful tool, both in modelling real-world problems and as a widely applicable mathematical theory. However, many interesting optimization problems are non linear. The studies of such problems involve a diverse blend of linear Algebra, multivariate calculus, numerical analysis and computing techniques.

The simplex method which is used to solve linear programming was developed by George B. Dantzig in 1947 as a product of his research work during World War II when he was working in the Pentagon with the Mil. Most linear programming problems are solved with this method. He extended his research work to solving problems of planning or scheduling dynamically overtime, particularly planning dynamically under uncertainty. Concentrating on the development and application of specific operations research techniques to determine the optimal choice among several courses of action, including the evaluation of specific numerical values (if required), we need to construct (or formulate) mathematical model (Hiller et al, 1995) (ADAMS,1969), (DANTZIG,1963) .

Conclusively, the development of linear programming has been ranked among the most important scientific advances of the mid-20th century, and its assessment is generally accepted. Its impact since 1950 has been extra ordinary. Today it is the standard tool that has saved thousand or million of dollars of many production companies.

Methodology and Data Analysis

The method to be adopted is the Revised Simplex Method for standard maximization problem. In order to use the method the following procedure is necessary:

- Introduction of slack or surplus variable if need be and bring the problem into standard form after converting the problem into maximization or minimization.
- Find an initial basic feasible solution with initial basic B = I_m (identity matrix) and form auxiliary matrix B such that:
  $\begin{bmatrix} B^{-1} & 0 \\ C_{BB}^{-1} & I \end{bmatrix} = B^{-1}$ and $\begin{bmatrix} B & 0 \\ -C_B & I \end{bmatrix}$ B =
- Considering the objective function as an additional constraint, form A and b such that:

  A = $\begin{bmatrix} A \\ -C \end{bmatrix}$ and b = $\begin{bmatrix} b \\ 0 \end{bmatrix}$
- Compute the net evaluation:
  . A$(C_{BB}^{-1} \;\; I)$ = C_j . Z_j

It should be noted that:

- If all Z_j - C_j $\geq$ 0, the current basic solution is an optimum solution.
- If at least one Z_j - C_j $<$ 0, determine the most negative, say Z_k - C_k corresponding to variables X_k enters the basis.
- Compute X_k = C_{curr}^{-1} . a_k ,also the solution is unbounded if X_k $\leq$ 0 and if at least one X_k $\leq$ 0, consider the X_m and determine the leaving variable.
- Write down the result obtained from steps 2 and 5 above in a revised simplex table.
- Convert the leaving element to unity and all other element of the column k to zero and improve the current basic feasible solutio
- Go to step 4 and repeat the procedure until an optimum basic feasible solution is obtained or an indication of an unbounded solution.

For the purpose of this research work the bye product of GEEPEE Nigeria Limited were used. There are four products produced in the company, this is stated as follows:

Figure 1:

NAME OF TANK	LITRES	GALLON	HEIGHT (mm)	CODE
COMBO	46000	10000	2600	2XWT
ATLAS	32000	7000	2900	2XWT
RAMBO	23000	5100	2600	WT23
JUMBO	16000	3500	1900	WT160C

Source: Field Survey October, 2010.

The data collected were as follows:

Figure 2:

NAME OF TANK	MATERIAL POLYETHYLENE (kg)	TIME (Min)	OXYACETYLENE (kg)	PROFIT (DOLLAR)
COMBO	14	6	20	60
ATLAS	12	4	15	45
RAMBO	12	3	15	30
JUMBO	10	3	12	27
MATERIALS AVAILABLE	97	42	100	

Source: Field Survey October, 2010.

The Linear Programming model is formulated as:

Max (Z) = $60x_1 + 45x_2 + 30x_3 + 27x_4$

Subject to:

$14x_1 + 12x_2 + 12x_3 + 10x_4 \leq 97$

$6x_1 + 4x_2 + 3x_3 + 3x_4 \leq 42$

$20x_1 + 15x_2 + 15x_3 + 12x_4 \leq 100$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$ for all non-negativity condition.

By introducing the slack variable in the objective functions above we have:

Max (Z) = $60x_1 + 45x_2 + 30x_3 + 27x_4 + 0S_1 + 0S_2 + 0S_3$

Subject to:

$14x_1 + 12x_2 + 12x_3 + 10x_4 + 0S_1 + 0S_2 + 0S_3 = 97$

$6x_1 + 4x_2 + 3x_3 + 3x_4 + 0S_1 + 0S_2 + 0S_3 = 42$

$20x_1 + 15x_2 + 15x_3 + 12x_4 + 0S_1 + 0S_2 + 0S_3 = 100$

For non negativity condition:

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$, $S_1 \geq 0, S_2 \geq 0, S_3 \geq 0$

The augmented matrix is obtained as:

$$A = \begin{bmatrix} 14 & 12 & 12 & 10 & 1 & 0 & 0 \\ 6 & 4 & 3 & 3 & 0 & 1 & 0 \\ 20 & 15 & 15 & 12 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 97 \\ 42 \\ 100 \end{bmatrix}$$

$$C = \begin{bmatrix} 60 & 45 & 30 & 27 & 0 & 0 & 0 \end{bmatrix}$$

C_b = [■(0&0&1)]

$$B_{curr}^{-1} = \begin{bmatrix} C_B & 0 \\ C_{BB}^{-1} & 1 \end{bmatrix}$$

$$B_{curr}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 14 & 12 & 12 & 10 & 1 & 0 & 0 \\ 6 & 4 & 3 & 3 & 0 & 1 & 0 \\ 20 & 15 & 15 & 12 & 0 & 0 & 1 \\ -60 & -45 & -30 & -27 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{b}{0} \end{bmatrix}$$

$$B = \begin{bmatrix} 97 \\ 42 \\ 100 \\ 0 \end{bmatrix}$$

To obtain the net evaluation (Z_j - C_j) we have:

$Z_j - C_j = (C_{BB}^{-1} \; I) . A$

$$= [(0 \; 0 \; 0 \quad 1)] \begin{bmatrix} 14 & 12 & 12 & 10 & 1 & 0 & 0 \\ 6 & 4 & 3 & 3 & 0 & 1 & 0 \\ 20 & 15 & 15 & 12 & 0 & 0 & 1 \\ -60 & -45 & -30 & -27 & 0 & 0 & 0 \end{bmatrix}$$

=[-60 -45 -30 -27 0 0 0]

In the net evaluation of Z_j - C_j is the highest negative, then x_1 enters the bases so that:

$X_1 = B_{curr}^{-1} . b$

$$X_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 97 \\ 42 \\ 100 \\ 0 \end{bmatrix}$$

$$X_b = \begin{bmatrix} 97 \\ 42 \\ 100 \\ 0 \end{bmatrix}$$

To draw the revised simplex tableau we have:

X	X_b	B_{curr}^{-1}	X_1	Ratio
S_1	97	1 0 0 0	14	6.929
S_2	42	0 1 0 0	6	7
S_3	100	0 0 1 0	20	5
Z	0	0 0 0 1	-60	0

1st Iteration

Converting the leading element to unity and all other, element to zero, we have:

$R_3' \rightarrow$ ($^1/_{20} R_3$) this becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 14 \\ 6 \\ 1 \\ -60 \end{bmatrix} = R_3'$$

To obtain the next net valuation $Z_j - C_j$ we have:

$$Z_j - C_j = [0\ \ 0\ \ 3\ \ 1] \begin{bmatrix} 14 & 12 & 12 & 10 & 1 & 0 & 0 \\ 6 & 4 & 3 & 3 & 0 & 1 & 0 \\ 20 & 15 & 15 & 12 & 0 & 0 & 1 \\ -60 & -45 & -30 & -27 & 0 & 0 & 0 \end{bmatrix}$$

$Z_3 - C_3$ = [0 0 15 9 0 0 3]

Since $Z_j - C_j \geq 0$, i.e. there is non-negative value in the net evaluation, the current feasible solution is optimum. Hence;

$X_b = B^{-1}_{curr} . b$

$$X_b = \begin{bmatrix} 1 & 0 & \frac{-7}{10} & 0 \\ 0 & 1 & \frac{-7}{10} & 0 \\ 0 & 0 & \frac{1}{20} & 0 \\ 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 97 \\ 42 \\ 100 \\ 0 \end{bmatrix}$$

$$X_b = \begin{bmatrix} 27 \\ -28 \\ 5 \\ 300 \end{bmatrix}$$

Finally $X_1 = 5$; $Z = 300$

Conclusion

The data collected from the industry on four types of Tanks produced, namely Combo, Atlas, Rambo and Jumbo was subjected to statistical analysis using the Revised Simplex Method. It was observed that if GEEPEE NIGERIA LIMITED can produce five units of Combo Tanks with an objective Coefficient of Sixty Dollars, it will give an objective value contribution of Three Hundred Dollars.

To this extent and based on the model formulated, and the analysis carried out that the amount of polyethylene and oxyacetylene used in the production of different sizes of plastic material produced with the allotted time attached to each product; Combo Tanks (of height 2,600mm, containing 4,600 litres i.e. 10,200 gallons of liquid) assures a profit margin of three hundred dollars if the quantity produced stood at 5 within the specified period of time.

Recommendatiion

After it has been established in the course of this research work that among other products, Combo Tanks assures more profit in GEEPEE NIGERIA LIMITED. We thereby recommend that:

The management of GEEPEEE NIGERIA LIMITED should give more attention to the production of the product (Combo) than other products because it seems to give the highest profit.

In the same vein, critical examination should be carried out on other products on their contribution to the growth or success of the company, if their profit margin is very low, then their production can be ignored.

Finally, any product having an adverse effect or contributing losses to the profit margin of the company can be stopped and the company invest more on the production of Combo to generate more profit.

References

Adams W. J. (1969): Element of Linear Programming . Van Nostrand Reinhold Publishing Company International.

Andrade, E. L. (1990): Introdução à Pesquisa Operacional. LTC, Rio de Janeiro.

Bierman Jr., Bonini H., & Charles P. (1973): Quantitative Analysis for Business Decisions. 4th edition, Richard D. Irwin, Illino

Corrar, Teophilo L., Carlos Renato (2003): Pesquisa Operacional para Decisão em Contabilidade e Administração. Editora Atlas, Rio de Janeiro.

Dantzig G. B. (1963) : Linear Programming and Extension, Priceton University Press.

David Charles. H. (1982): Introduction Concept and Application of Operation Research

Hiller, F.S., G.J. Lieberman and G. Liebeman (1995): Introduction to Operations Research, New York: McGraw-Hill

Nearing E.D and Tucker A.W. (1993): Linear Programming and Related Problems. Academic Press Boston.