Research Article

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Towards Learner-Centered Teaching: The Case of Mathematics Teachers

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Abstract

Following the results of the TIMSS survey qualifying the school crisis, the Moroccan education system in its reform has launched the "Learner-Centered Teaching" (LCT) model which is part of a plan which aims to linking the classroom space to the two school spaces on the one hand and the socio-economic environment on the other hand, and this, through a new teaching practice that exploits the pedagogical and didactic elements of the competency-based approach, of interdisciplinarity, project-based pedagogy and institutional communication. This article presents the training adopted for the benefit of teachers within the framework of the LCT according to four professional situations whose competence aimed at the end of the training is to manage, in terms of learning, mathematical concepts by taking into account the mathematics education resources and tools.

Keywords: *teaching practice, didactics, problem solving, learner-centered teaching, mathematics*

1. Introduction

The results of the TIMSS 2019 survey involved the participation of 64 countries and nearly 580,000 students. Morocco participated for the third consecutive time (2011, 2015, 2019) in this survey with 8,830 fourth-grade primary school students. It is consistently ranked in the fourth group of countries with an average below 400 in mathematics (below the international average of 500). In order to dispel certain prejudices and negative judgments arising from this ranking, the Ministry of Education has launched a plan for the implementation of the 2022-2026 reform. The objective of this plan is to ensure excellent practical and professional training for education and training professionals, particularly mathematics teachers. This plan was implemented in three pilot regions, targeting a total of 36,000 teachers. for a period of 3 years of experimentation, the educational inspectors highlighted in their reports the positive impact of the approach on the daily lives of teachers in terms of the content taught. This convinced the Ministry of Education to seriously think about generalizing this approach throughout the country. Subsequently, a comprehensive training program was conducted for trainers and educational inspectors, lasting five days, with the aim of replicating it for all teachers across the country. The "Learner-Centered Teaching" (**LCT**) model, which is a part of the mentioned plan, aims to establish a link between the classroom and both the school environment and the socio-economic context. This is accomplished through the implementation of a new teaching approach that utilizes the pedagogical and didactic foundations of competency-based learning, interdisciplinary integration, project-based pedagogy, and institutional communication. Teacher training within the framework of **LCT** is offered through four professional situations, with the targeted competency at the end of the training being the ability to effectively manage the learning of mathematical concepts while incorporating mathematics didactics resources and tools.

This approach differs from traditional teaching methods because it pushes the teacher to make choices about the traditionally proposed situations and to choose the most relevant ones, thus serving the construction of the meaning of mathematical objects among students, and this, by mobilizing other teaching methods such as the student's personal project, the interdisciplinary aspect of mathematics and digital technologies. The approach thus proposed provides, on a micro-didactic level, tools which enable the teacher to analyze learning situations on various aspects : linguistics, semiotics, mathematical organization and mathematical representation systems that have potential impact on student learning outcomes.

Due to administrative and logistical constraints related to educational personnel, this training was conducted through three face-to-face professional situations and a fourth one in a distance learning mode. We retrospectively reconstructed the actual implementation process in various workshops to achieve the optimal outcomes for the participants.

In short, to summarize with a didactical engineery the scientific methods used in this study, we present a schematic representation of the steps involved in conducting the study witch allows us to tackle the program of teacher training within the framework of LCT:

Step 1. Proposed implementation: In this paragraph, the training schedule is presented in the form of three professional situations which bring together the following seven tasks :

Task 1: Objectify one's representations, experiences, and needs regarding the skill of constructing a situation related to **LCT**;

Task 2: Analyze and explain the characteristics of a **LCT** and the tools for constructing a **LCT**;

Task 3: Analyze a didactic situation that promotes **LCT** and the different stages of such a situation;

Task 4: Construct a complex situation;

Task 5: Consider semiotic representation systems in mathematics education;

Task 6: Analyze regarding questions of **ICTE** use;

Task 7: Elaborate a project-based learning situation that aligns with learner-centered teaching and takes into account the 3 professionalization spaces.

Step 2. Results : We present synthesis of the work of groups relating to each of the seven tasks above.

Step 3. Discussion : We give analysis of teachers' productions.

2. Proposed Implementation

We suggest that the training be carried out during a non-working session (e.g., school holidays), allowing for optimal utilization of small group work. The groups were initially formed arbitrarily during the first session, with the possibility of changing groups for the second day of training. As for the training scenario, it generally consists of four key stages:

- Lecture presentation (duration: 20 to 30 minutes)
- Group work activities (duration: 30 minutes to 1 hour and 20 minutes)
- Feedback and sharing session (duration: 40 minutes to 1 hour)

- Synthesis (duration:20 to 30 minutes)

Below, we present the sequence of each of the three professional situations (PS) divided into an average of one PS per day, consisting of two training sessions. Each PS is composed of tasks. We provide an overview of the content and resources utilized by the facilitator for each task, followed by conducting analyses based on the results of the teachers' productions.

Table 1. Training Schedule

Days	Professional situations (PS)	Sessions and Tasks	
Day 1	PS 1	**Session 1**	Task 1- Task2
		Session 2	Task 3
Day 2	PS 2	**Session 1**	Task 4
		Session 2	Task 5
Day 3	PS 3	**Session 1**	Task 6
		Session 2	Task 7

3. Professional Situation 1: How to characterize a situation related to LCT in mathematics and with which analysis tools?

The first professional situation (**SP1**) is composed of three tasks in which the trainees are asked to answer the following question:

How can we characterize a situation related to **LCT** in mathematics and with which analysis tools? These three tasks are the following:

Task 1: Objectify one's representations, experiences, and needs regarding the skill of constructing a situation related to **LCT**.

Task 2: Analyze and explain the characteristics of a **LCT** and the tools for constructing a **LCT**.

Task 3: Analyze a didactic situation that promotes **LCT** and the different stages of such a situation.

Regarding Task 1: Objectify one's representations, experiences, and needs regarding the skill of constructing a situation related to **LCT**.

Many of us share the idea that there is often a resistance among teachers to change their daily practice from one day to the next! In **Task 1**, we want to objectify teachers' representations of the concept of **LCT**. We also want to invite them to identify and exploit their own experiences in relation to **LCT**. Finally, we want to convince them of the necessity of the change in the learning paradigm required by the competence of constructing an **LCT** situation.

Methodologically, the Q-Sort technique is suggested in order to align mental conceptions with our didactic considerations related to the concept of **LCT**.

Our Q-Sort is proposed in the form of 20 statements approaching the targeted **LCT** approach. The teacher must individually classify them in orderly categories according to what he/she considers more or less good for him/her. Then by interaction between the members of each group a single grid is given to the facilitator. We will try to correlate the teachers based on these rankings.

3.1 Statement for Q-Sort on Learner-Centered Teaching

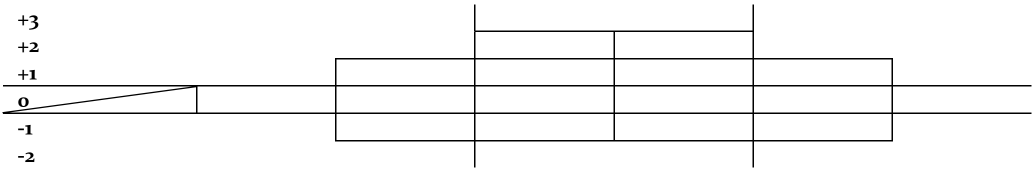
Sentences:

1. Something that helps the learner succeed in their class exams.
2. An initiative that improves the quality of the learner's learning.
3. A more useful approach for other disciplines than mathematics.
4. A waste of time that prevents the timely completion of the curriculum.
5. A method to integrate cross-curricular and interdisciplinary teachings.
6. An approach that should be proposed by the ministry, similar to curriculum frameworks.

7. Teaching that takes into account the learners' levels.
8. Most useful for experienced teachers.
9. Useful for beginner teachers.
10. Something sufficient to be a good teacher.
11. Revamping the organization of teaching sessions.
12. A process that should be consistently ensured by the teacher.
13. Teaching that takes into account the socio-economic dimension of the learner.
14. A process that should be occasionally ensured by the teacher.
15. Teaching that takes into account the differences among learners.
16. Teaching that takes into account the school's project.
17. A process that promotes project-based pedagogy.
18. Teaching that requires a solid foundation in mathematics education.
19. Teaching that engages new learning situations.
20. An approach that promotes working with complex situations.

Sentence numbers.

Table 2. Q-Sort on Learner-centered Teaching



3.2 *Results and Analysis of Q-Sort*

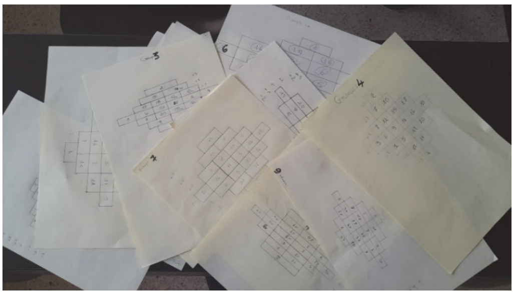


Figure 1. Examples of observed responses.

phrase	Groupe 1	Groupe 2	Groupe 3	Groupe 4	Groupe 5	Groupe 6	Groupe 7	total
phrase 1	0	-1	0	-1	1	0	1	0
phrase 2	1	2	3	3	3	-1	1	12
phrase 3	-2	0	-2	0	-1	-1	0	-6
phrase 4	-3	-2	-2	-3	-3	-3	-1	-17
phrase 5	0	3	0	-1	0	0	-3	-1
phrase 6	2	1	0	0	1	-1	1	4
phrase 7	3	-3	0	2	2	2	2	8
phrase 8	-1	-1	0	1	-1	1	-1	-2
phrase 9	0	0	-1	0	1	-2	-1	-3
phrase 10	0	0	2	2	0	-2	-2	0
phrase 11	-1	0	1	-1	0	0	0	-1
phrase 12	-2	0	-1	1	0	0	0	-2
phrase 13	0	2	-3	-2	0	0	1	-2
phrase 14	-1	-1	-1	0	-1	0	0	-4
phrase 15	2	0	1	1	2	3	2	11
phrase 16	-1	1	0	0	0	1	0	1
phrase 17	1	-2	2	-2	1	2	-2	0
phrase 18	0	1	1	1	-1	1	-1	2
phrase 19	1	1	1	0	-2	1	3	5
phrase 20	1	-1	-1	-1	-2	-1	0	-5

Figure 2. The two most frequently cited sentences in the Q-Sort related to LCT.

From the figure above, it is observed that unanimously, there is a consensus that **LCT** would improve the quality of the learner's learning (sentence 2) and that this pedagogical model is beneficial without causing a loss of time in the teaching-learning process (sentence 4). The presentation of this result has created a positive atmosphere for exchange, and we see the beginnings of a fruitful training!

Regarding Task 2: Analyze and explain the characteristics of a **LCT** and the tools for constructing a **LCT**.

For this task, the facilitator presents an initial presentation to initiate the trainees' reflection on what it means to "give meaning to learning" (Perrenoud, 1996; Develay, 1996; Douady, 1986; Charnay, 1999; Bouvier 1981). We want to open the eyes of teachers to the fact that **LCT** requires from the outset a clear differentiation between the two paradigms of teaching and learning (Legendre, 2001). In this case, in the evaluation and analysis framework of the PISA survey (2015, p.71), it is indicated that mathematical culture provides a significant complement which is the link to society and to human history. It is this aspect that makes it meaningful. Mathematics is not learned essentially for its own sake, but to understand and solve the problems of the world. In order to provide the teacher with the didactic basis for characterizing learning situations, the presenter gave a second presentation on the typology of problems. A first classification of the types of exercises (Bodin, 2003). Then other classifications according to the expected level of mathematization (PISA, 2019), according to the activities and processes (Régis Gras, 1974, 1975), according to the different functions of problems. The last two slides of this second presentation, in our opinion, require particular attention, the first one is also presented on the characteristics of a problem-situation according to Douady (1986) while the second one introduces the concept ZPD (Proximal Development Zone) according to Vygostki (1997).

4. Results and Analysis of Teachers' Productions

During the opening presentation for the benefit of teachers, the idea was highlighted that the targeted core competency of the **LCT** project could be developed through an integrated approach across three learning spaces, namely the classroom, the school, and the socio-economic environment. In the assigned task regarding the characteristics of a situation related to **LCT**, this driving idea generated descriptors that were found to be redundant in the trainees' responses (see Figure 3).

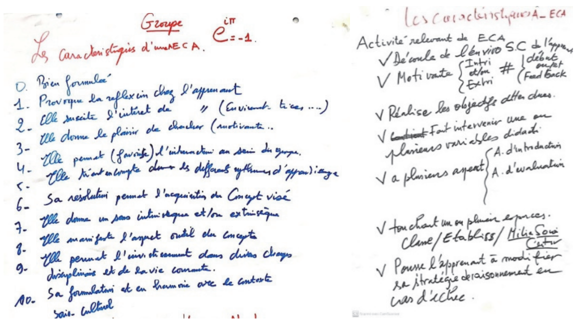


Figure 3. Examples of workshop-day 1 productions.

The majority of trainees tend to provide descriptors that are generally associated with the three learning spaces related to **LCT** instead of perceiving the characteristics of a learning situation. A particular emphasis is placed on the motivational dimension, where expressions such as "it arouses interest in…," "it provides the joy of exploration…," "intrinsically and/or extrinsically motivating" are frequently used.

We assume that characterizing a learning situation would require a good understanding of didactic analysis tools, which is the objective of the next task.

Regarding task 3: Analyzing a didactic situation that promotes a **LCT** and the different moments of such situation.

A thorough analysis of a learning situation involves both a priori analysis, which means anticipating what the learner will do in the face of a mathematical activity in terms of procedures, obstacles, and the application of targeted knowledge. On the other hand, a posteriori analysis focuses on the connection between the mathematical task and the results obtained.

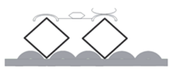
In three parts, the facilitator begins by delivering a presentation on the theoretical foundations that can shed light on didactic analysis tools, while revisiting the concept of "concept," this time within the framework of the Theory of Conceptual Fields (**TCF**) proposed by Gérard Vergnaud in 1990. This cognitivist theory aims to provide basic principles for studying the development and learning of complex skills, particularly the "schema" defined as the invariant organization of the learner's behaviors for a class of learning situations.

A second component focuses on the theory developed by Guy Brousseau (1973), known as the Theory of Didactic Situations in Mathematics (**TDSM**). This part specifically explores the concepts of "didactic variable" and "didactic contract." Finally, a third component introduces a proposed framework for a priori analysis of a learning situation.

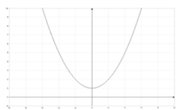
In a second phase, the facilitator distributed the Situation1 1 below (Figure 4) to the participants. The goal was to conduct a priori analysis using a simplified analysis framework consisting of three categories of questions.

SITUATION 1: "Chain curves"

Part 1: A fun challenge... Can you ride a bicycle with square wheels?



Part 2: The graph below represents the curve of a numerical function defined over the set of real numbers in a Cartesian coordinate system.:



1. What can you say about the continuity and monotonicity of this function?
2. Does the proposed function have a reciprocal function on the entire set of real numbers or only on a subset?
3. Calculate the images, if they exist, by the reciprocal function of the points with abscissas zero, two, negative one, and negative three.

Part 3: Let's consider the numerical function defined on IR by: $b(x)=\frac{e^{-x}+e^{x}}{2}$

1. Study the numerical function b on its domain, and then plot its graph in a Cartesian coordinate system.
2. Let f be the restriction of the function b to $[0;1000000]$. Show that f has a reciprocal function defined on an interval to be determined.
3. Complete the following table:

	The function f	The reciprocal function
Definition set		
Continuity		
Monotony		
The tangents		
The asymptotes		

4. Deduce the graph representation of the reciprocal function. f^{-1}
5. Calculate the expression of the reciprocal function.

Figure 4. Statement of Situation 1 for prior analysis

[1] *Activity proposed by Salek OUAILAL.*

4.1 *Results and analysis of teachers' productions*

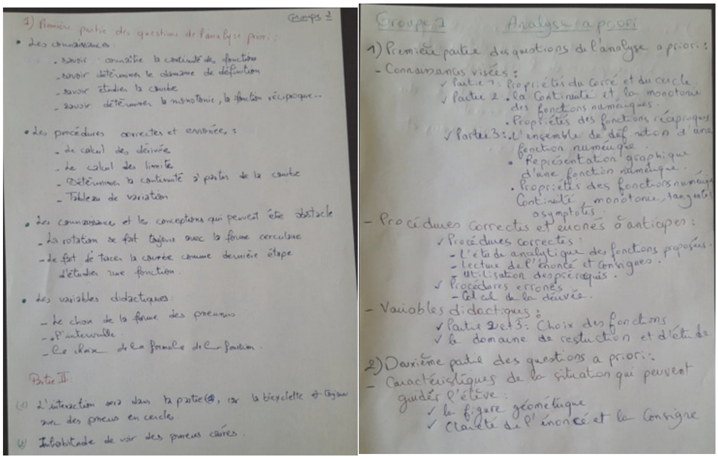


Figure 5. Examples of observed responses.

The written work of the participants shows, in general, the existence of two groups in terms of didactic writing methodology. The first group successfully presented the written formulation of the analysis elements (the targeted grade level, the objectives of the situation, the knowledge, the skills to be highlighted for solving the situation, the possible difficulties that students may face in this situation). The second group - the majority - only wrote keywords of mathematical objects mentioned in the situation (see Figure 5). We explain this by the fact that the teachers in the first group are candidates who have recently taken the Secondary Education Teaching Aptitude Test (**APES**) or have prepared for the entrance examination for the training cycles at the Center for Teacher Inspector Training (**CFIE**). Considering this divergence, the facilitator presents a priori analysis and a proposed sequence for each part of the given situation (see Table 3), discussing various aspects related to **LCT**.

Table 3. A priori analysis and proposed process

Didactic commentary		**In terms of LCT**
Part 1	The first part consists of a single question that has a concrete meaning for the students. The discussion of the answer could be done first within each group, and then through interaction between the groups. In this case, the brainstorming technique is suggested in order to generate original ideas. The teacher keeps the solution[2] until the end of the session. The goal is, on one hand, to break away from the routine of the classroom, and on the other hand, to keep the students' attention throughout the activity as each of them will be intellectually engaged in deciphering the challenge.	Promoting the aspect of mathematics derived from everyday life.
Part 2	In the second part, we proposed a "simple" graph (the parabola) well-known to high school students. Additionally, since they can easily approach the questions asked in this part, the objective is to give a sense of controllability to the exercise. The text does not contain any mathematical symbols; we simply used a statement in natural language to invite the learner to reflect on the meaning and use of mathematical symbolism, as well as the semiotic representation register.	Working within a semiotic representation register

[2] *The reader can see the video on: https://youtu.be/FlvjWpWu99A*

Part 3	The third part is a topic that students are not accustomed to working on: filling in a table, using the letter "b" instead of "f" for function notation, writing the "large" number 1000000 instead of +∞. The objective is twofold: firstly, to provide students with opportunities to develop formalistic thinking, initiating the transition to formal mathematics at the higher level, and secondly, to emphasize the graphical tools necessary for constructing the graph of a numerical function, as working within a graphical framework is often less emphasized in the classroom.	Developing formalistic thinking Mobilizing frame shifting
Part 4	The teacher can ask the students to plot, using dynamic geometry software, the graphs of the functions b and x^2 + 1 on the same coordinate plane. These graphs tend to have a similar shape (see Fig. 4), which should help the learner visualize the underlying convexity. In this context, an interesting didactic variable is to change the values of the constant C in the expression of the function B(x) = [exp(-Cx) + exp(Cx)]/2, which is responsible for the convexity of the graph.	Utilizing ICT tools

4.2 *Synthesis of the small groups' works related to the professional situation 1.*

Under the premise that there is no predefined definition of what is Learner Centered Teaching (**LCT**) and after having disserted a certain set of didactic concepts related to mathematical activity in the classroom in the light of the teachers' experiences. We present in the table below the highlights from the three tasks performed on the first day of the training:

Table 4. General concepts of the first day of training

LCT context	• Results of Moroccan students in international evaluations (PISA, TIMSS); • The necessity to change teaching practices in the mathematics class in the view of the contemporary mutations of the Moroccan educational system (change of teaching language, renewal of human resources by recruitment of thousands of new contractual teachers); • The tendency of educational-collaborative work that the new Moroccan school demonstrates through the installation of the Integrated School Project (ISP).
LCT Definition	A pedagogical model that stimulates the teacher's reflection in order to guide their practices towards a learning paradigm that takes into consideration the learner's interests in relation to the 7 specific competences[3] of mathematics teaching.
LCT Characteristics	A situation relevant to **LCT** should: • Be proposed in the form of a problem situation with the aim of mobilizing the learner's knowledges. • Involve the interdisciplinary dimension as an exploitation domain. • Be based on real life. • Motivating and stimulating the learner's interest. • Reach out to other learning spaces (school, socio-economic environment).

5. Professional Situation 2: Constructing a LCT Related to a Problem Point

In order to achieve our fundamental objective of developing a learning situation related to **LCT** while considering the three learning spaces (Task 7), the facilitator presents two presentations on theoretical and methodological tools, namely: the characteristics of a complex situation, the significance of a situation (De Ketele, Chastrette, Cros, Mettelin & Thomas, 1989, p. 100); emblematic situation (Perrenoud, 2001); language (Ouailal, 2015); semiotic representation systems in mathematics education (Duval, 1988, 1993); and frame change (Douady, 1986). The second professional situation (PS2) consists of two tasks :

Task 4: Construct a complex situation

Task 5: Consider semiotic representation systems in mathematics education.

[3] *The reader will have access to the reference text, which was published in August 2009 by the Curriculum and School Life Directorate, addressing the programs and pedagogical guidelines pertaining to the teaching of mathematics in the secondary education cycle.*

5.1 Results and analysis of teachers' productions

Regarding task 4: Construct a complex situation.

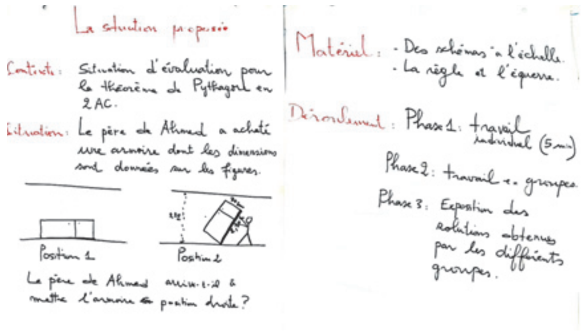


Figure 6. An example of workshop-day 2 production.

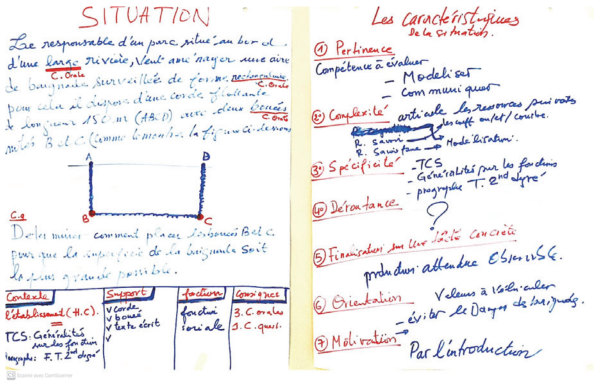


Figure 7. An example of workshop-day 2 production.

The analysis of the productions of the small groups (see Figure 3 and 4) demonstrates similarities in the productions performed with a lot of oral exchanges, especially in the dimension related to the third socio-cultural learning space. The material distributed to the teachers on the characteristics of a complex situation allowed them to go into more finely detail in the analysis of the mathematical situations conceived in terms of relevance, complexity, specificity, confusion, orientation, and motivation. If the exchange and discussion were rich from a didactic point of view, it is particularly remarkable that the statements of conceptions contained texts, which is very rare in the Moroccan mathematics classroom, this language aspect also favoured the gender approach and a certain sensitivity to some social values.

Regarding task 5: Taking into account registers of semiotic representation.

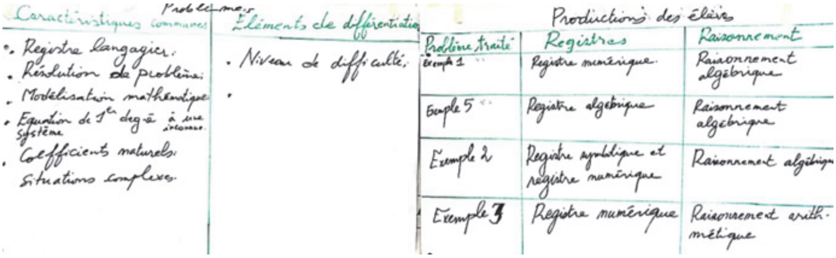


Figure 8. An example of workshops- day 2 production.

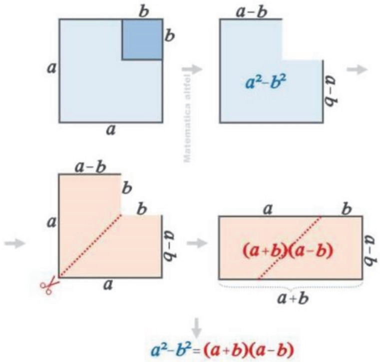


Figure 9. An example of workshops – day 2 production

Initially, this task to be carried out is far more complex because the beneficiaries have not fully grasped the distinction between the two concepts, namely « framework » and « register », and it requires taking a step back regarding these concepts. To clarify the ideas, the facilitator presented the table below, aiming for a clearer reformulation with some illustrations!

Table 5. Illustrations of key concepts in Task 5

Frameworks	We say that we work within a given framework if we study a situation whose data, statements, and primary study tools are situated within a well-defined main theory. A framework thus appears as a field of work. We can invoke: • The framework of analysis (continuity properties, differentiability...). • The set-theoretic framework (applications, equations, injectivity and surjectivity, inverse functions, inverse images...). • The algebraic framework (types of functions and the formulas defining and governing them: polynomials, circulars...). • The geometric framework (graphs of functions with specific geometric properties: lines, parabolas and hyperbolas, questions of asymptotes, of convexity...). • The functional analysis framework (which is also within the set-theoretic framework: functions become elements of sets of functions, we have applications between sets of functions, functional equations...) ; • The numerical analysis framework (approximation and computational methods).
Registers	Registers are modes of representation of mathematical objects, accompanied by processing rules, which allow to study problems: For example, numerical functions have four main registers of representation. • Register of value tables. • Register of formulas. • Register of graphical representation. • Symbolic register (with an implicit set-theoretic framework).

The facilitator therefore distributed student productions for analysis in terms of register (Figure 8) and also asked the groups to propose situations that promote a change of frameworks (Figure 9).

6. Professional Situation 3: Conducting project-based learning across the three learning spaces

To implement a situation that promotes learner- centered teaching (**LCT**), this PS3 presents the information and communication technologies in education (**ICTE**) as an operational tool. The first objective is to train teachers in handling the softwares that serve to build school mathematical concepts. Semiotic representation issues are also addressed in this aspect. The second is to exploit the training corpus to design project-based learning devices. This PS3 is proposed in two tasks:

Task 6: Analyze regarding questions of **ICTE** use.

Task 7: Elaborate a project-based learning situation that aligns with learner-centered teaching and takes into account the 3 professionalization spaces.

6.1 Results and analysis of teacher's productions

Regarding Task 6: Analyze on questions about the use of **ICT**.

The concept of ICT is quite familiar to all teachers. In the initial phase, the facilitator asked the participants, based on their teaching-learning experiences, to fill in a table with three columns, listing software programs according to three dimensions: addressing a mathematical concept, making mathematical conjectures, practicing and/or remedying a mathematical error (Figure 10).

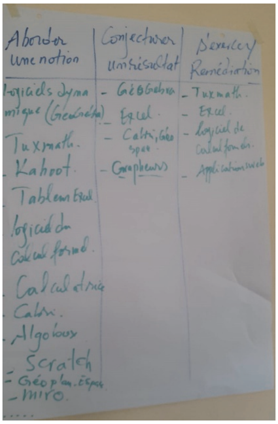


Figure 10. An example of workshop-day 3 productions.

Secondly, the facilitator assigned the situation[4] below (Figure 11) to the beneficiaries, the objective was to exploit computer tools to facilitate the change of framework and register in order to understand an optimization phenomenon:

[4] *Activity proposed by Alain Bronner.*

AB is an arc of a circle with center O.
M is a point on the segment [OA].
The perpendicular to (OA) at N intersects the arc AB at M.
Determine, if it exists, one or more positions of point M for which the area of triangle OMN is maximum.

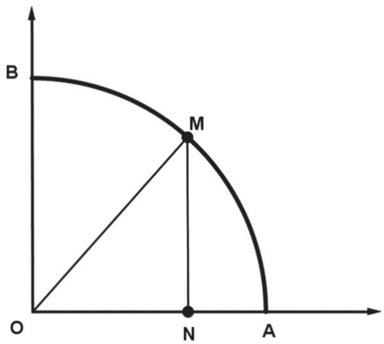


Figure 11. Optimization situation from the perspective of changing frameworks and registers

The small groups consist of teachers with heterogeneous levels of proficiency in using dynamic geometry software. To promote exchange between "experts" and "beginners" in Geogebra software, the facilitator has reorganized the groups. Figure 9 clearly illustrates the respective manipulation of the geometric, graphical, and numerical frameworks. While some have adopted a formal approach by working within an algebraic framework, using equations to solve the problem and then verifying the experimental results.

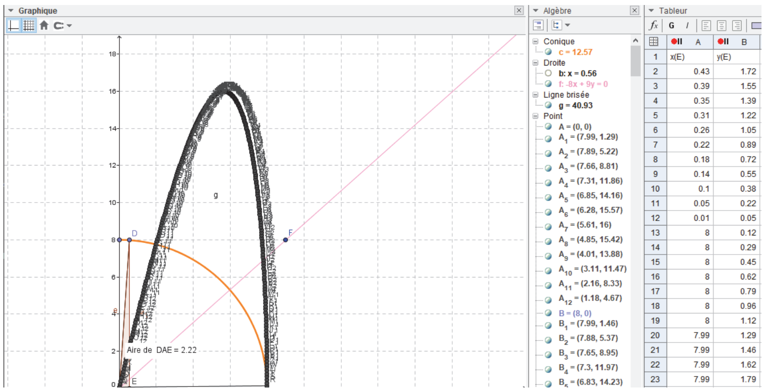


Figure 12. An example of production from workshop-day 3.

Regarding Task 7: Elaboration of a project-based learning situation aligned with learner-centered teaching and considering the 3 professionalization spaces.

For this last sequence, we believe that we have provided teachers with didactic vocabulary that can encourage them to steer their teaching practice towards an innovative model, namely learner-centered teaching (LCT), using an integrative approach to the three learning spaces (classroom, school, environment). The instruction given by the facilitator is to design an LCT learning framework, and as a recap, they have reiterated the following training concepts :

Table 6. Illustrations of the selected training concepts.

Involvement of learners in an ECA model;
Meaning of mathematics;
Innovation;
Interdisciplinarity;
Semiotic registers: symbolic, formal, natural language, formal language, algebraic, geometric, etc.;
Mathematization of situations;
Coordination between different registers;
Integration of ICT;
Openness to the learner's socio-educational environment;
Project-based approach (PBA).

The more LCT-rich tasks are proposed, the more significant the productions and exchanges of the beneficiaries appear. We must note that, for the majority of the groups, the conceptions of LCT-related situations mainly involve interdisciplinary aspects, the use of ICT, and mathematics Olympiads. The table below presents the emerging aspects with examples of teachers' productions.

Table 7. Example of situations fostering LCT according to the three learning spaces.

Aspect	Learning Space	Illustration
ICTE	class	Figure 15
Mathematics Olympiad	Institution	Figure 14
interdisciplinarity	Environment	Figure 13

Situation 2. Trigonometry and navigation at sea.
One of the ancient applications of trigonometry is the principle of triangulation, which allows determining a location based on two other locations. It is a fundamental tool for navigation at sea. To obtain a measure of the distance "d," Thales of Miletus (around 625-620 BC - 548-545 BC) placed two observers, A and B, on the shore, separated by a known distance "l." He asked each of them to measure the angle formed by the lines passing through the boat and one of them, and the line passing through the two observers.

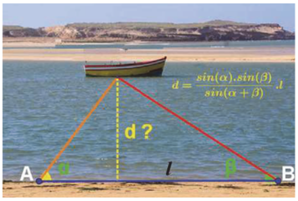


Figure 13. Statement of Situation 2.

Situation 4 : A useful inequality for mathematical Olympiads

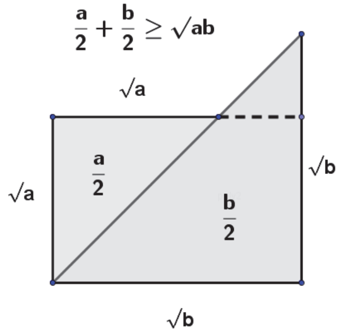


Figure 14. Statement of Situation 3.

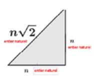
Situation 4. Irrationality of the square root of two in a geometric context.

A gardener has a measuring rod that measures 1 meter. He would like to place trees evenly spaced on the front portion of his garden (See figure).

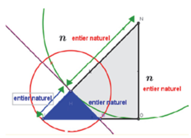
1) Draw isosceles and right triangles with respective sides measuring 1cm, 2cm, and 3cm.

2) Calculate the lengths of the hypotenuses of each of the previous triangles.
3) Two hypotheses are given:

H1: The length of the hypotenuse of a triangle of this type is a natural number.



H2: There exists the smallest triangle of the same nature.
See the figure above.



What can be inferred?
4) In your opinion, is the gardener's project feasible?

Figure 15. Statement of Situation 4.

7. Conclusion

The context in which this research was conducted is set within the framework of the continuous professional development project for educational leaders implemented by the Moroccan education system. The aim is to enhance the skills of teachers and innovate their teaching practices. The approach adopted, known as **LCT** (Learner-Centered Teaching), aims to achieve a 100-point advancement in the TIMSS ranking by 2030.

Through the use of small-group work, we have attempted to trigger an awareness of the contribution of mathematics didactics to the innovation of teaching practices. This was accomplished through three professional situations encompassing seven tasks.

At the end of this training, the teacher should be able to implement learning situations by considering mathematics didactics tools. Specifically, they should be able to manage a Learner-Centered Teaching (**LCT**) approach, integrating the three learning spaces (class, institution, and environment). In terms of evaluation, after returning to their classrooms, the beneficiaries will be interviewed about how they have used their knowledge in their teaching. A report will be requested to demonstrate how the teacher has applied their **LCT** training. It is proposed that teachers capitalize on their training progress by maintaining a portfolio. However, we believe that a reflective teacher who conducts research on their own teaching practices is the key to any successful educational reform.

8. Limitations of Study

1. Our sample is quite limited and may not even be considered a representative sample of teachers in Morocco;
2. Difficulty of programming continuing training for teachers at certificate school levels;
3. Lack of digital infrastructure in some educational establishments;
4. Hesitation of some teachers to participate in the establishment's collective projects;
5. The training program is disrupted by any possible strike of teachers
6. The study was limited to the scholastic year 2023-2024.

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