



Research Article

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## Exploring Primary School Mathematics Teachers' Strategies for Enhancing Students' Mathematical Writing Skills

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### Abstract

*This study aimed to determine female mathematics teachers' level of performance in employing the teaching practices necessary to develop mathematical writing among female primary school students. It used the descriptive survey approach and collected data through observation. The study sample comprised 48 female teachers. The study generated several interesting results, most notably that the mathematics teachers' level of performance was average, with overall means of 1.92–3, and there were statistically significant differences at the level of  $\alpha < .05$  in the level of teaching practices employed attributable to teaching experience in favor of the group with fewer than 10 years' experience. Based on these results, the study makes several recommendations. Primarily, it proposes that training programs for in-service female mathematics teachers should be strengthened, ensuring they cover the teaching practices necessary to develop mathematical writing skills.*

**Keywords:** mathematics; teaching practices; mathematical writing; primary school stage

### 1. Introduction

Mathematics is an essential medium for exchanging ideas clearly and accurately as it is a language that can be used to communicate regardless of differences in the mother tongue. Developing mathematical writing is one of the most essential objectives of teaching mathematics. Indeed, the National Council of Teachers of Mathematics (NCTM, 2000) Principles and Standards for School

Mathematics cite this as one of the key mathematical communication skills and a requirement for the processes of mathematical correlation and reasoning necessary to build conceptual and procedural mathematical knowledge and solve problems.

Mathematical writing is a tool for presenting mathematics in linguistic form that includes symbols, signs, pictures, drawings, and agreed-upon terms used to build and convey mathematical meaning, enhance communication, and express thinking. Mathematical language consists of (1) symbols of numbers, zero, forms, and dimensions, (2) terms signifying concepts such as the arithmetic mean, standard deviation, distance, velocity, function, and inequality, with each term having a symbol that expresses it, and (3) mental and visual images, including graphs, tables, shapes, and charts. Mathematical writing is used in expression, interpretation, and proof (Byung-In, 2015).

Mathematical writing is represented in symbolic translation, writing mathematical proofs in sequence and logically, writing the mathematical inferences used, and justifying thought paths to produce and apply mathematical knowledge. Bani Araba and Al-Ghafri (2012) define mathematical writing as the student's ability to use the language of written mathematics, its terminology, and symbols to express mathematical thinking processes and explain and clarify mathematical ideas, including expression through drawings and tables, quantitative expression using numbers and units of measurement, symbolic expression using algebraic magnitudes, relationships, functions, equations, inequalities, and expression using suitable algorithms when writing mathematical proofs. Byung-In (2015) defines it as a symbolic system that conveys meaning, and this meaning depends on its context. For example, the subtraction symbol (-) without context represents a sign, while  $(23-15 = 21)$  is a mathematical sentence that denotes subtraction and a thinking activity that signifies removing part of the whole. Obeida (2018) defines it as the student's use of mathematical language symbols and terminology to express mental operations when solving mathematical problems and providing proofs. It includes the following skills: Symbolic expression (using mathematical symbols to accurately express a term, form, concept, and generalization), explanation and justification (writing and explaining algorithms and abstractions used to solve problems clearly in a logical sequence), and mathematical proof (a written expression of understanding mathematical data and using it in proving a theory according to the sequence and interdependence between the premises and the required data and the use of mathematical logic (Alneyadi et al., 2023).

According to Chen and Raley (2013) mathematical writing denotes students expressing ideas, mental paths, and preferences for thinking patterns using mathematical symbols. Based on the aforementioned information, mathematical writing can be defined as a student's proficiency in articulating their mathematical knowledge, encompassing both conceptual understanding and procedural skills. It involves effectively utilizing symbols, laws, and theories relevant to the given context in order to solve mathematical problems. Santos and Cernana (2015) contend that it is related to the symbolic interpretation of thinking, and its skills are represented in writing explanations, justifications, and mathematical proofs. For instance, the skill of mathematical writing in algebra is related to using symbols and variables to describe mathematical relations, deduce laws and properties, express unknowns, and interpret algorithms. In geometry, it expresses figures and dimensions using symbols, writing their characteristics, deducing theories, and writing mathematical proofs. Casa et al. (2020) identifies the sub-skills of mathematical writing as follows: teaching performance related to enhancing the paths of multiple solutions for students, teaching performance related to encouraging students to describe and explain their answers, teaching performance related to encouraging students to share their reasoning and mathematical proofs, and teaching performance related to encouraging students to consider the validity of a given solution and discuss the validity of two given solutions (Alneyadi et al., 2023).

Mathematical writing serves as a means for students to assimilate and construct mathematical knowledge, aligning with the constructivist theory's educational philosophy that highlights the learner's active role. Additionally, mathematical writing facilitates the learning process and encourages verbal and written expression of mathematical ideas (Adam, 2017). To foster this, it is crucial for teachers to create sports experiences and activities that enable and motivate students to engage in mathematical writing (AlAli et al., 2023).

Written tasks and activities contribute to understanding-based learning by providing students with appropriate opportunities to question and inquire and write what they have understood in their way. Thus, they allow students to build their mathematical knowledge individually, think about their mathematical writing, and correlate it with their thinking throughout all the stages of writing. Moreover, in the process of thinking about what they write and how they present their ideas in writing, they consider how to harmonize their previous and new experiences and understandings in a written form that helps them develop a special meaning (Abu Ar-Rayat, 2019). Mathematical writing also helps students develop positive attitudes toward mathematics (Teuscher, 2015). It contributes to deepening the absorption of mathematical knowledge and improves teaching and learning outcomes because it reflects the extent of students' learning and their ability to express it clearly (Kenny, 2014). Moreover, mathematical writing supports mathematical communication skills and encourages the use of metacognitive skills through working on mathematical problem-solving tasks (Knox, 2017).

With a firm belief in the significance of education and the pivotal role of teachers in directing the educational process towards its goals, educational institutions worldwide have placed utmost importance on the teacher. They are deemed responsible for the success or failure of any educational system as the primary element in the educational process. The teacher's effectiveness and influence play a crucial role in determining the quality of educational outcomes. Despite advancements in curricula and the availability of technology and educational resources, teachers remain essential in utilizing these capabilities to enhance educational outcomes and achieve the objectives of the educational process (Obaid, 2004).

The "Learning is the Treasure Within" report highlights the significance of shaping education as a key aspect of the twenty-first century. It identifies four main pillars: learning to know, learning to work, learning to live with others, and learning to prove oneself. The report emphasizes the active role of teachers in the educational process and the necessity of enhancing their training and working conditions to meet the demands of the modern era (UNESCO, 1996). The educational conference "Educational Leadership: Contemporary Visions" (2016) stresses the importance of empowering teachers and transforming their role from knowledge transmitters to leaders and mentors. It calls for the thoughtful integration of educational skills, curricula, and practical solutions in the classroom (Tashtoush et al., 2023).

For mathematics teachers to effectively assume their new educational roles, which focus on creating a conducive learning environment and managing it in a manner that fosters student engagement and positivity, they must possess the necessary teaching skills in their specialized field. Furthermore, they should be able to practice and apply these skills to ensure the success of the educational process.

Mathematics teachers can enhance students' mathematical writing skills through several steps (Powell, 2016). First, the teacher assigns the students a task, asking them to write down what they understand concerning the meaning and significance of a new concept. In this step, they will also express the relationship between the new mathematical concept and concepts learned previously by drawing a diagram or a conceptual map. Second, the teacher asks the students to compare their ideas and solutions with their peers. Each pair of students discusses the relevance of these ideas to the solution. They identify the common ideas and support them with evidence and proof endorsing their point of view. Third, the teacher asks them to rewrite their ideas about appropriate solutions. They amend or develop their written ideas in light of what they learned from their colleagues. Fourth, the students combine all their ideas for the solution to come up with new and appropriate proposals to solve the mathematical problem at hand. Fifth, each student presents a summary of his/her ideas and proposed solution to the mathematical problem to their classmates. They discuss this together to clarify and justify the solution. It is essential to present and discuss all the mathematical ideas and solutions that the students came up with.

In this regard, some studies have investigated the level of mathematical writing skills and related practices among mathematics teachers. For example, As-Suraihi (2022) investigated the level of performance of primary school mathematics teachers in implementing mathematical writing skills in class. The study found that from the perspective of educational supervisors in Madinah, the teachers' performance was average with reference to Education and Training Evaluation Commission's specialized standards framework, specifically the standard of using mathematical

communication and employing related skills in teaching mathematics. Ash-Sharif and Qandil (2020) also found that supervisors reported that the mathematical writing skills of mathematics teachers were average. Rayani and An-Nafish's (2019) study showed average mathematical writing skills among male and female teachers in the Sharurah Governorate and also found no statistically significant differences attributable to the effect of years of service on skills. Abdullah and Jaafar's (2017) study identified above average teaching performance and writing skills among mathematics teachers, with a positive correlation between the level of teaching performance and mathematical writing skills. Moreover, they found a statistically significant difference between the mean scores of mathematics teachers for teaching performance and the skill of mathematical writing attributable to experience in favor of five or more years of experience. In a different context, Sür and Delice (2016) found that mathematics teachers in Turkey depend on mathematical writing skills during teaching.

There are many advantages of mathematical writing. For example, it enables students to organize their mathematical ideas and opinions to develop solutions in an appropriate and mathematically correct written manner. It helps learners self-evaluate their performance by identifying their strengths and weakness and supporting them in addressing the latter. It provides students with appropriate opportunities to elucidate the arguments and proofs underpinning mathematical ideas (Van Dyck & Heffer, 2014). It also helps teachers to see students' mathematical cognitive structures and the strengths and weaknesses in their performance, so that they can improve students' ability to complete written mathematical tasks and employ the students' writing to build new mathematical ideas for them (Firmender et al., 2017).

Moreover, it provides students with real opportunities to participate actively in mathematical cognitive activities, enhances their self-responsibility in performing mathematical tasks, and motivates introverted students who refrain from oral expression to express themselves as learners of mathematics in writing and describe the extent of their understanding of mathematical experiences (Casa et al., 2020). In addition, it provides an opportunity to develop students' metacognitive thinking skills. Students summon and organize their ideas and express them in writing, which enables them to think about their ideas, re-evaluate and develop them, and clarify the evidence and proofs that support their validity (Gadong et al., 2016)

One of the advantages of mathematical writing is that it enables students to think about the mathematical problem by referring to their prior knowledge of mathematical concepts and relationships related to its solution, apply appropriate methods and strategies to solve it, and organize their ideas to explain and clarify each step in solving it. The importance of mathematical writing is evident in that it allows students to think deeply about the ideas presented to solve mathematical problems. Mathematical writing affords a slower thought process than oral expression and students who write out new mathematical ideas they have learned tend to be more accurate than those who do not. Therefore, writing is an essential factor that strengthens students' learning of mathematics and builds strong foundations (Bani Araba & Al-Ghafri, 2012). Describing in writing how to solve a mathematical problem helps students clarify their thinking and deepens their understanding. Students can also read their peers' writing and explain their thinking about the content and aims of what is written. Evaluating students' competence in solving problems convincingly and supporting their solutions with evidence based on justifications and illustrative examples of the ideas and generalizations provides many opportunities for students to improve their writing and for teachers to determine their students' skills in organized thinking (Bostiga, 2016).

Moreover, developing students' mathematical writing skills is critical as these skills are needed to derive mathematical proof and use mathematical knowledge in solving problems. Mathematical writing requires students to progress from oral representation and proof to written confirmation using generalizations and providing justifications of algorithms and mathematical ideas (Filiz & Ozdemir, 2014). Mathematical writing skills are also clearly important as mathematics is an approach to logical thinking. Mathematical writing supports students' in producing mathematical ideas and organizing their mathematical thinking (Broto & Greer, 2014). Obeida (2018) suggested that the importance of developing mathematical writing among students is evident because it enhances writing students' thinking paths,

which then supports the development of new paths. It also supports students in identifying incorrect algorithms and inappropriate strategies and fosters understanding of the logical order of the steps in deriving solutions through the application of writing strategies for solving mathematical problems. Furthermore, it encourages students to produce, record, discuss, and evaluate mathematical ideas and reinforces the required skills, such as reading, representing, and speaking mathematically.

According to Quealy (2014), the significance of written mathematics as an introduction to learning mathematics stems from the fact that it connects course topics and applies concepts. Furthermore, it helps students justify their thinking patterns, apply the steps in finding the solution logically, build inferences, and comprehend and produce knowledge. It can also be employed in diagnosing students' mathematical misconceptions. Walk (2010) showed that effective teaching of mathematics motivates students to think and speak in the language of mathematics, then use mathematical writing to enhance and organize their thinking paths, and comprehend the meaning of terms, ratios, and quantities. In addition, it is an essential process in learning mathematics and transitioning from oral expression of mental operations to written expression.

Mathematical writing helps students reflect on their thinking when they solve problems, as it allows them to process information accurately and review the mathematical concepts studied. It also allows them to build their knowledge and express their mathematical understanding. Students can practice their reasoning and think in depth when expressing their ideas; this contributes to meaningful learning, deepens students' understanding of the topic, and enriches their mathematical learning (Guce, 2017).

Written mathematics activities also help students reflect on the activities they are assigned and express their mathematical beliefs and ideas freely. Al-Kkidr and Ar-Rasheedi (2021) identified several mathematical writing types: writing journals (students are asked to write about the concepts and notions that have been presented to them and their understanding of them), free writing (students are assigned certain tasks without being limited to a specific manner of writing about them), and reflective writing (students have the opportunity to express in writing their thoughts and feelings on a topic).

Given the importance of mathematical writing, many studies have sought to use it as a teaching input to develop various skills, such as the ability to solve mathematical problems and mathematical thinking. For instance, Čutura and Vulović (2020) concluded that there is a relationship between the development of mathematical thinking and mathematical writing activities in that they require students to formulate mathematical problems and specify the data, the required steps in arriving at the solution. Al-Hadhrameyyah and Al-Abed (2016) considered that mathematical writing increases students' awareness of how to learn and remember, provides a record of students' thinking, offers tools for writing ideas in multiple ways and summarizing previous learning, and provides a continuous record of the challenges that students face during learning. Gadong et al. (2013) also found that mathematical writing enhances students' metacognitive skills, their ability to express their attitude toward what they learn, and the ability to understand mathematical content and recall previous knowledge, thus leading to meaningful learning. Thus, mathematical writing is a good tool for assessing students' conceptual understanding. It addresses the cognitive and affective aspects of assessment that traditional assessment methods may overlook because it provides evidence of the type and level of students' conceptual and procedural learning. It is a means of understanding students' learning of specific mathematical concepts. Al-Badawi and Abu Loum (2013) pointed to the role of mathematical writing in helping students show their existing knowledge and clarify their thinking in solving mathematical problems, enabling them to present their ideas and opinions in writing, developing their ability to communicate mathematically by asking questions and discussing their ideas collaboratively, and evaluating and modifying them in concert with others. Parsons (2011) showed that reflective mathematical writing helps students comprehend and learn mathematical concepts by involving them in forming a meaning for the concept, which has a profound and meaningful educational impact. Al-Khadhour and Al-Ghafri (2010) recommended training students to write mathematical proofs logically using deductive relations; this would guide students in how to write the data and the required steps arrange a set of statements and explanations in a piece of evidence, complete a partial proof, deduce a result from a given statement, justify this result, and write a multi-step proof with or without a guiding idea.

## 2. Research Problem

Although mathematical writing is essential for achieving the objectives of learning mathematics, building thinking skills, problem solving, and giving mathematical proof, and it plays a role in providing the teacher with the opportunity to evaluate students' comprehension and organization of their thinking, many studies have found out students have difficulties in mathematical writing skills and using the language of mathematics. They lack learning and communication skills, and the ability to write algorithms and solve problems. This affects their thinking and mathematical achievement levels. For instance, Khalil (2015) showed that primary school students have low skills in writing, representing describing and interpreting mathematical symbols, and expressing mathematical concepts and relations. Obeida's study (2018) showed weaknesses among students in writing mathematical proofs and interpreting the mathematical generalization used in the solution, as well as their ability to justify their answers, and formulate and translate mathematical problems from verbal images to symbolic images. Fukawa (2012) found that students could present justifications and proofs verbally but faced challenges presenting them in writing. Writing requires strategies for encoding the mathematical problem, using symbols, terms, graphs, and tables, distinguishing data and the required steps and writing the theory underpinning solutions. It also requires logical thinking skills to comprehend the relations between the steps in arriving at solutions, provide justifications, and distinguish between the given information and the results.

In terms of the reasons for deficits in mathematical writing skills, Martin (2015) found that low levels of such skills can be attributed to the students' low levels of language. To address this, the teacher must reinforce the writing of symbols, terms, and concepts, and explain the problem-solving writing plan and the associated algorithms. Asha et al. (2014) found that low levels of mathematical writing skills among students, despite the availability of mathematical writing activities in textbooks, was due to deficiencies in the teachers' pedagogical practices and strategies. Obeida (2018) showed that the development of students' mathematical writing skills is primarily based on teachers' pedagogical practices and that teachers must repeatedly train students in mathematical dialogue to develop clarity of ideas, solutions, and paths of thinking, then initiate communicating in writing using a problem-solving plan and related generalizations before working on the solution and encourage students to interpret algorithms. Because of the importance of mathematical writing as a dimension of mathematical communication and as a means of teaching and learning mathematics, there is increased interest in measuring teachers' pedagogical practices that support its development among students. This study addresses the need for such research.

## 3. The Study

### 3.1 Research Objectives and Questions

The study sought to achieve the following objectives:

To identify the level of teaching performance of female mathematics teachers in the use of teaching practices necessary to develop mathematical writing among female primary school students.

To establish the extent to which there are statistically significant differences at the level  $\alpha < .05$  between the average scores of mathematics teachers in implementing the teaching practices necessary to develop mathematical writing attributable to length of teaching experience.

To address these objectives, the study sought to answer the following questions:

1. What level of performance do female mathematics teachers present in implementing the teaching practices necessary to develop mathematical writing for primary school students?

Are there statistically significant differences at the level  $\alpha < .05$  between the mean scores of female mathematics teachers for the teaching practices necessary to develop mathematical writing attributable to length of teaching experience?

### 3.2 Conceptual Framework

This study was designed to investigate four aspects of the performance of mathematics teachers in employing teaching practices to develop mathematical writing among primary school students. These four aspects were as follows:

Dimension 1: Teaching performance related to the promotion of multiple solution pathways for students.

Dimension 2: Teaching performance related to encouraging students to describe and explain their solutions.

Dimension 3: Teaching performance related to encouraging students to share their mathematical reasoning and proofs.

Dimension 4: Teaching performance related to encouraging students to consider the validity of a given solution and discuss the validity of two given solutions.

The items associated with these four aspects are presented in the conceptual framework in Figure 1.

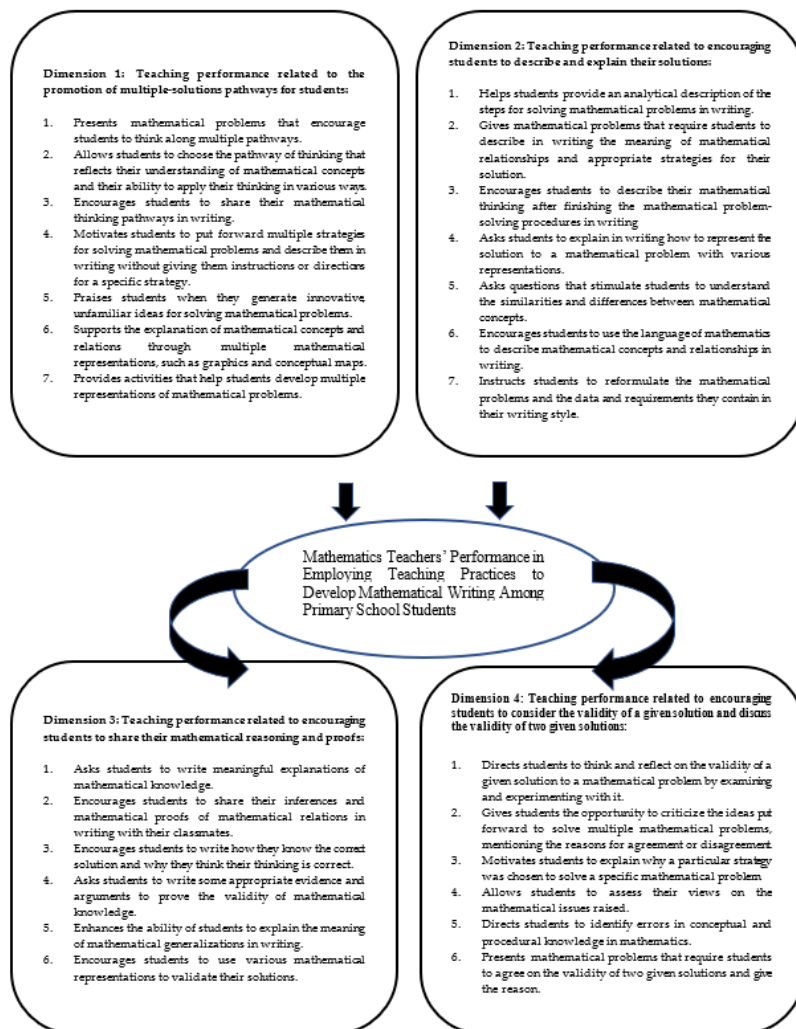


Figure 1. Conceptual Framework of the Performance of Mathematics Teachers in Employing Teaching Practices to Develop Mathematical Writing Among Primary School Students.

#### 4. Methodology

The study employed the descriptive survey method, commensurate with the study's aim to identify the performance level of female mathematics teachers in employing the teaching practices necessary to develop mathematical writing among primary school students.

##### 4.1 Population and Sample

The study population consisted of female primary school mathematics teachers in the Bisha Education Department (N = 178) in the academic year 2021/2022. The study sample comprised 48 female teachers, representing 27% of the study population. The selection process for the participants in the study was conducted using a stratified random sampling method. The educational areas under the jurisdiction of the Bisha Education Department were first identified. From each of these areas, a random sample was selected to participate in the study. This approach ensured that the participants were representative of the various educational areas in the region while also maintaining a random selection process within each area. Table 1 shows the distribution of the respondents according to years of teaching experience.

**Table 1.** Distribution of Respondents Based on Years of Teaching Experience.

Experience	Frequency	Percentage
≤ 10 years	26	54.2
> 10 years	22	45.8
Total	48	100

##### 4.2 Study Instrument

The study employed observation card, which aimed to identify the level of female teachers' performance in implementing the teaching practices necessary to develop mathematical writing for primary school female students. The steps in preparing the card were as follows:

Consulting the educational literature and previous relevant studies to determine the dimensions of mathematical writing skills that represented the axes of the observation card. The most important reference was the study of Casa et al. (2022). They are: enhancing multiple solution paths, encouraging students to describe and explain their answers, urging students to share their mathematical reasoning and proofs, prompting students to consider the validity of a given solution, and fostering discussions on the validity of two provided solutions.

Writing a set of appropriate classroom teaching practices for each dimension.

Calculating the validity of the study tool: The study tool was assessed for its validity by presenting it to a group of experts and specialists in mathematics education. Their input was sought to evaluate the tool in terms of the appropriateness of its components and the consistency of teaching practices with the main dimensions. Additionally, their feedback was considered to determine the feasibility of observing these dimensions.

Calculating the reliability of the instrument using the observers' agreement method. This involved trialing the observation card by observing and evaluating four female mathematics teachers' practices and calculating the coefficient of agreement between the observers using Cooper's equation. Table 2 shows the percentages of agreement.

**Table 2.** Observers' Level of Agreement Indicating the Reliability of the Observation Card.

Teacher 1	Teacher 2	Teacher 3	Teacher 4	Overall agreement
70%	88.5%	92.3%	85%	84%



Formulating the tool in its final form. After verifying validity and reliability, the final form of the observation card included the following sections: (i) Respondent data: name (optional), years of teaching experience; (ii) General instructions stating the purpose of the observation card, how to use it to express the teacher's level of teaching practices, and assuring the confidentiality of data to be used only for scientific research purposes; (iii) Dimensions of the observation.

The dimensions and number of teaching practices in each were as follows:

Dimension 1: Enhancing the multiple solution paths for female students (7 teaching practices).

Dimension 2: Encouraging female students to describe and explain their answers (7 teaching practices).

Dimension 3: Encouraging female students to share their mathematical reasoning and proof (6 teaching practices).

Dimension 4: Encouraging female students to consider the validity of a given solution and discuss the validity of two given solutions (6 teaching practices).

The observation card incorporated a three-level scale to evaluate the teachers' performance in employing the teaching practices for each dimension: 3 = high, 2 = medium, and 1 = low. Then the length of the interval was calculated as follows:  $3-1 = 2$ ,  $2/3 = 0.6$ . Table 3 shows the distribution of categories according to the gradation used in the observation card.

**Table 3.** Distribution of categories according to the gradation used in the observation card

Instrument	Performance level	Range of means
Observation card	High	2.44-3
	Medium	1.67-2.43
	Low	1-1.66

### Application procedures:

Following the necessary approvals from the Bisha Education Department, the evaluation process was planned in collaboration with the educational supervisor. The supervisor attended complete regular classes for each teacher and assessed their performance using the observation card. None of the cards were excluded as all the required conditions for answering them were met. Consequently, a total of 48 observation cards were included in the study.

## 5. Results

### 5.1 Level of female mathematics teachers' performance in employing teaching practices necessary to develop mathematical writing among primary school female students

To address the first research question, we calculated the means and standard deviations for each statement. Table 4 shows the results.

**Table 4.** Level of Female Mathematics Teachers' Implementation of Teaching Practices Necessary to Develop Mathematical Writing Among Primary School Students.

No.	Practice	Mean	SD	Level
<b>Dimension 1: Teaching performance related to the promotion of multiple solution pathways for students</b>				
1	Presents mathematical problems that encourage students to think along multiple pathways	2.10	.592	Medium
2	Allows students to choose the pathway of thinking that reflects their understanding of mathematical concepts and their ability to apply their thinking in various ways	2.13	.672	Medium
3	Encourages students to share their mathematical thinking pathways in writing	1.90	.0857	Medium
4	Motivates students to put forward multiple strategies for solving mathematical problems and describe them in writing without giving them instructions or directions for a specific strategy	1.79	.849	Medium
5	Praises students when they generate innovative, unfamiliar ideas for solving mathematical problems	2.35	.758	Medium
6	Supports the explanation of mathematical concepts and relations with multiple mathematical representations such as graphics and conceptual maps	2.15	.849	Medium
7	Provides activities that help students develop multiple representations of mathematical problems	1.98	.799	Medium
Overall performance		2.06		Medium

No.	Practice	Mean	SD	Level
<b>Dimension 2: Teaching performance related to encouraging students to describe and explain their solutions</b>				
8	Helps students provide an analytical description of the steps for solving mathematical problems in writing	1.90	.905	Medium
9	Gives mathematical problems that require students to describe in writing the meaning of mathematical relationships and appropriate strategies for their solution	1.85	.865	Medium
10	Encourages students to describe their mathematical thinking after finishing the mathematical problem-solving procedures in writing	1.85	.899	Medium
11	Asks students to explain in writing how to represent the solution to a mathematical problem with various representations	1.88	.914	Medium
12	Asks questions that stimulate students to understand the similarities and differences between mathematical concepts	1.90	.857	Medium
13	Encourages students to use the language of mathematics to describe mathematical concepts and relationships in writing	1.88	.941	Medium
14	Instructs students to reformulate the mathematical problems and the data and requirements they contain in their writing style	1.88	.914	Medium
Overall performance		1.88		Medium
<b>Dimension 3: Teaching performance related to encouraging students to share their mathematical reasoning and proofs</b>				
15	Asks students to write meaningful explanations of mathematical knowledge	1.83	.834	Medium
16	Encourages students to share their inferences and mathematical proofs of mathematical relations in writing with their classmates	1.81	.842	Medium
17	Encourages students to write how they know the correct solution and why they think their thinking is correct	1.88	.914	Medium
18	Asks students to write some appropriate evidence and arguments to prove the validity of mathematical knowledge	1.83	.883	Medium
19	Enhances the ability of students to explain the meaning of mathematical generalizations in writing	1.79	.898	Medium
20	Encourages students to use various mathematical representations to validate their solutions	Medium	.895	Average
Overall performance		1.84		Medium
<b>Dimension 4: Teaching performance related to encouraging students to consider the validity of a given solution and discuss the validity of two given solutions:</b>				
21	Directs students to think and reflect on the validity of a given solution to a mathematical problem by examining and experimenting with it	1.90	.857	Medium
22	Provides e students with the opportunity to criticize the ideas put forward to solve multiple mathematical problems, mentioning the reasons for agreement or disagreement	1.92	.895	Medium
23	Motivates students to explain why a particular strategy was chosen to solve a specific mathematical problem	1.90	.831	Medium
24	Allows students to assess their views on the mathematical issues raised	1.90	.865	Medium
25	Directs students to identify errors in conceptual and procedural knowledge in mathematics	1.88	.831	Medium
26	Presents mathematical problems that require students to agree on the validity of two given solutions, along with mentioning the reason	1.83	.864	Medium
Overall performance		1.89		Medium
Performance as a whole		1.92		Medium

The results shown in Table 4 demonstrate that the overall performance of female mathematics teachers in implementing the teaching practices necessary to develop mathematical writing among primary school female students was at the medium level, with a mean performance value of 1.92. Also, the performance for each axis was medium. The mean value for performance on Dimension 1 was 2.06 with a range in the mean values for the implementation of practices of 1.79–2.35. The practice “Praises students when they generate innovative, unfamiliar ideas for solving mathematical problems” attained the highest mean score, while the practice “Motivates students to put forward multiple strategies for solving mathematical problems and describe them in writing without giving them instructions or directions for a specific strategy” received the lowest mean score, but all the practices within this dimension were at the medium level.

Next, for Dimension 2, the overall mean for performance was 1.89, with mean scores for individual practices in the range 1.83–1.92. The practice “Provides students with the opportunity to criticize the ideas put forward to solve multiple mathematical problems, mentioning the reasons for agreement or disagreement” attained the highest mean score, whereas the practice “Presents mathematical problems that require students to agree on the validity of two given solutions, along with mentioning the reason” presented the lowest mean for performance. However, all practices within this dimension were at the medium level.

The overall mean value for performance on Dimension 3 was 1.88, with a range in the means for practices from 1.85 to 1.90. The highest mean scores were for the two practices “Helps students provide an analytical description of the steps for solving mathematical problems in writing” and “Asks questions that stimulate students to understand the similarities and differences between mathematical concepts.” In contrast, the lowest mean scores were for the two practices “Gives mathematical problems that require students to describe in writing the meaning of mathematical relations and appropriate strategies for their solution” and “Encourages students to describe their mathematical thinking after finishing the mathematical problem-solving procedures in writing”.

However, all practices within this dimension were at the medium level.

Finally, for Dimension 4, the overall mean was 1.84 and with mean scores for individual practices in the range 1.79–1.92. Among these, the practice “Encourages students to use various mathematical representations to validate their solutions” had the highest mean value for performance, while the practice “Enhances the ability of students to explain the meaning of mathematical generalizations in writing” had the lowest. All practices within this dimension were at the medium level.

### 5.2 Differences in the mean scores for mathematics teachers in implementing the teaching practices necessary to develop mathematical writing attributable to teaching experience

The second research question addressed whether there were differences in teachers’ practices according to their teaching experience that were statistically significant at the  $\alpha < .05$  level. We used the t-test for the calculation and Table 5 shows the results.

**Table 5.** T-Test Results of the Differences in Teaching Practices Attributable to Teaching Experience.

Variable	Category	Number	Mean	SD	t-value	Significance
Teaching experience	< 10 years	26	61.00	17.68	5.242	.00
	≥ 10 years	22	35.95	13.31		

The results in Table 5 show a t-value of 5.242, which is statistically significant at the level of 0.00. This indicates that there are statistically significant differences at the level  $\alpha < .05$  between the mean scores of female mathematics teachers employing the teaching practices necessary to develop mathematical writing attributable to teaching experience in favor of teaching experience of < 10 years ( $M = 61.00$  against  $M = 36.95$  for  $\geq 10$  years).

## 6. Discussion and Conclusion

The results for the first research question showed an average level of performance of female mathematics teachers in implementing the teaching practices necessary to develop mathematical writing among primary school female students, with a mean score of 1.92. The level of performance along each of the axes and for each practice was also average. This result is consistent with the findings of Al-Surayhi (2022), Abdullah and Jaafar (2017), and Sür and Delice (2016). It could be attributed to female teachers’ scientific background or their level of mathematical writing, in line with Rayani and An-Nafish (2019) and Ash-Sharif and Qandil (2020), who found that mathematics teachers had average mathematical writing skills, and Abdullah and Jaafar (2017), who identified a positive correlation between the level of teaching performance and mathematical writing skills among mathematics teachers. Moreover, this result may be related to female mathematics teachers’ beliefs about their students and about mathematical writing skills, or to the limited training programs offered to teachers in this field.

Modern mathematics curricula are rich in activities and problems that require higher-order thinking skills, including analysis, discussion, comparison, justification, evaluation, and presentation of multiple written ideas and solutions. This is one of the reasons in support of the need to foster the teaching practices required to develop mathematical writing. However, the intensity of mathematical content, the abundance of skills involved in learning mathematics, and limited class time could have restricted the ability of the female mathematics teachers to implement these practices properly, which would likely then negatively affect the level of female students’ mathematical learning. Obeid (2018), Asha et al. (2014) and Martin (2015) concluded that the low level of mathematical writing skills among students was due to deficiencies in teachers’ pedagogical practices and the teaching strategies they use, and these would require change to enable teachers to support the development of

mathematical writing skills. Al-Badawi and Abu Loum (2013), Al-Hadrameyyah and Al-Abed (2016), Al-Khadhour and Al-Ghafri (2010), Čutura and Vulović (2021), Early Sol et al. (2013) and Parsons (2011) emphasized the importance of employing teaching practices based on mathematical writing for developing students' achievement and other mathematical skills.

The results for the second question showed statistically significant differences at the level  $\alpha < .05$  between the average scores of female mathematics teachers in implementing the teaching practices necessary to develop mathematical writing attributable to teaching experience, in favor of  $< 10$  years teaching experience. This result differs from that of Rayani and An-Nafish (2019), who found no statistically significant differences attributable to the effect of years of service on skills, and also Abdullah and Jaafar (2017), who found a statistically significant difference between the mean scores of mathematics teachers for teaching performance and mathematical writing skill attributable to experience in favor of 5 years or more. This result can be attributed to the fact that female teachers with less experience have more recently completed their university studies, and programs and courses are continuously developed in light of recent trends. This may also be due to the focus of vocational training programs (in-service) on new and less experienced female teachers aimed at developing their expertise and supporting their teaching practices to meet the requirement of the curriculum and advances in the educational field. Moreover, female teachers with less than 10 years of teaching experience may be subject to professional competency tests and/or professional licensing as a necessary condition for work in the educational field and for professional promotion. Likewise, technological developments and the availability of applications and programs that support professional development may have a positive impact on the practices of less experienced female teachers, given their interest in technology and their ability to deal with it more easily than the older generation. This is supported by the findings of Al-Asmari and Ash-Shehri (2022), who found statistically significant differences in teachers' intention to use technology in their teaching attributable to years of experience in favor of  $< 10$  years.

## 7. Recommendations and Future Studies

In view of the results, this study makes several recommendations. First, it is necessary to augment the in-service training programs offered to mathematics teachers to develop the teaching practices required to improve mathematical writing skills. Such programs should incorporate advancements in the field of teaching and learning mathematics. This should be followed up with the application of skills in field training. Second, it would be of value to institute professional learning societies and encourage the exchange of experiences between female mathematics teachers with different teaching experience. Education departments should direct educational supervision practices to observe and measure the performance of female mathematics teachers and determine the on female students' learning of mathematical writing skills.

With regard to future studies, it would be of value to study the training needs of female mathematics teachers to develop the teaching practices necessary to foster the mathematical writing skills of primary school students. Studies could then offer a proposed training program based on constructivist theory to develop these teaching practices. Finally, studies could evaluate pre-service preparation programs for female mathematics teachers in terms of their treatment of mathematical communication skills.

## 8. Limitations

This study was limited to quantitative observation to assess teaching practices and specific strategies. Future work might employ both quantitative and qualitative tools. Performance was evaluated using a scale specifically developed by the researchers to measure the average overall performance for each exercise. The study was also confined to female teachers only. Other studies might encompass a broader sample.

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## 10. Institutional Review Board Statement

The study was conducted in accordance with the Declaration of Helsinki, and approved by the Ethics Committee of University of Bisha, Saudi Arabia protocol code ERS\_2022\_8789 and date of approval was 11 October 2022.

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