E-ISSN 2240-0524
ISSN 2239-978X

Journal of Educational and Social Research
www.richtmann.org

Vol 13 No 3
May 2023

Research Article

Received: 6 December 2022 / Accepted: 19 April 2023 / Published: 5 May 2023

Teaching and Learning of Mathematical Modeling for Hepatitis B Virus Transmission

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DOI: https://doi.org/10.36941/jesr-2023-0071

Abstract

This paper deals with the teaching and learning of mathematical modeling of hepatitis B virus (HBV) transmission. We first present the teaching of infectious diseases and more precisely the disease of hepatitis B in the Moroccan curriculum. In addition, we develop various activities in order to build the three forms of an HBV epidemic model that are visual, continuous and discrete. These activities can develop in students many abilities and skills such as mathematical reasoning, complex problem solving, modelling and communication. Furthermore, the construction of this HBV epidemic model will be done through the ordinary differential equations (ODEs) which are programmed in the teaching of mathematics in high Moroccan school.

Keywords: *Hepatitis B virus, infectious diseases, mathematical modeling, mathematics education, learning activities*

1. Introduction

Hepatitis B is a serious liver disease caused by the hepatitis B virus (HBV) that gives rise to both acute and chronic hepatitis. HBV spreads through direct contact with an infected person's blood, semen, or other body fluids (Chang, 2007). It can also be transmitted from mother to child during birth and delivery (Libbus and Phillips, 2009). Further, HBV is one of the most contagious and deadly viruses that infects and kills many people each year. For instance, World Health Organization (WHO) estimated that 296 million people were living with chronic hepatitis B infection in 2019, with 1.5 million new infections each year, and more than 820000 deaths in 2019, mostly from cirrhosis and liver cancer (WHO, 2022). A study carried out by Sbai et al. (Sbai et al., 2012) indicated that the prevalence of HBV in Morocco was estimated in 2012 at 1.66% in the active population.

Mathematical modeling plays a crucial role in understanding and describing the transmission dynamics of infectious diseases such as HBV. It forecasts the transmission trends of the disease, determines the key factors that influence its spread and establishes optimal strategies for disease control and prevention. Therefore, many researchers have discovered mathematical models describing the evolution of HBV within a population (Anderson and May, 1991; Khan et al., 2017; Williams et al., 1996; Kermack and Mackendrick, 1927; Hattaf et al., 2013; Din et al., 2020; Din et al., 2022). In addition, within a human body (in particularly, liver) (Hattaf and Yousfi, 2015; Hattaf and Manna, 2020; Manna and Hattaf, 2022; Bachraoui et al., 2021). However, there are few articles in the literature that focus on teaching and learning the mathematical modeling of infectious diseases (Harraq et al., 2020; Gaff et al., 2011; Lofgren et al., 2016).

In Morocco, baccalaureate students in the physics and chemistry option as well as the life and earth sciences option have the same mathematics program. However, most of the activities and exercises for mathematical modeling focus on the physics that interest students in the first option. In this paper, we focus on teaching and learning the mathematical modeling of infectious diseases especially hepatitis B through a simple and effective method based on students' prerequisites in both mathematics and life sciences scholar subjects. Such prerequisites are the knowledge and skills that a learner must master sufficiently and beforehand to be able to begin to study a new concept and/or acquire new skills.

The structure of the rest of this paper is as follows. The next section is devoted to materials and methods. Section 3 focuses on results and discussion including the HBV in Moroccan education system from primary to high school, as well as the mathematical modeling of HBV transmission dynamics through five learning activities that aim to build the visual, discrete and continuous version of an HBV epidemic model. Finally, Section 4 provides a conclusion of the study.

2. Materials and Methods

In this work, we have adopted a methodology based on two axes:

- Analysis of official texts related to the teaching of life and earth sciences in Morocco from primary to high school.
- Construction of learning activities that apply mathematical modeling to the transmission of hepatitis B in a given population.

For the first axis, we are interested in the lessons of life and earth sciences which deal with infectious diseases and especially hepatitis B disease. The research was based on the educational orientations and the official Moroccan curricula of the three cycles: primary school, middle school, and high school. For the second axis, in order to connect the teaching of the two disciplines: mathematics and biology, we have proposed mathematical modeling activities which integrate biology and especially the disease of hepatitis B. We took advantage of the mathematics lessons taught in the 2nd year of the baccalaureate, in particular differential equations and numerical sequences.

3. Results and Discussion

This section is devoted to results and discussion. We first focus on HBV in education system.

3.1 HBV in Moroccan education system

Since hepatitis B caused by HBV is an infectious disease, the aim of this subsection is to present in general the teaching and the learning of infectious diseases in Moroccan education system. Based on the Moroccan national charter of education and training (Royaume du Maroc, 1999), this education system includes preschool of two years, primary school of six years, and middle school of three years and high school of three years. The preschool is available to all children under the age of 6 years. Primary education is compulsory for all children between the ages of 6 and 11 years. The middle school receives students who have succeed in the exam of the sixth year of primary school. However, the high school is accessible to students who successfully passed the final exam of the third year of middle school.

According to (Royaume du Maroc, 2021), the new primary education curriculum is divided into three fields that are languages; socialization and openness as well as mathematics, sciences and technology. The last one consists of three subjects:

1. Mathematics that includes the components of numbers, arithmetic, measurement, geometry, organization and data processing.
2. Scientific activity that includes components of life sciences, physical sciences, earth and space sciences, and technology.
3. Informatics that based on activities using information technology and communication aimed at enabling learners to learn computer programming, create their own gaming applications and participate in targeted development projects.

Infectious diseases are taught in the subject of scientific activity, through the axis human health and interaction with the environment. The content of this axis for each year of primary school is summarized in Table 1.

Table 1: The content of the axis human health and interaction with the environment related to infectious diseases for each year of primary school.

Level	Subject	Content
First year	I protect my health	• Conclude that to be healthy good, he has to feed and keep his body clean. • Distinguish between state of good health and state of illness.
Second year	Nutrition and health	• Explain the importance of food hygiene for health. • Respect eating times. • Conclude the role of sports in maintaining healthy joints.
Third year	Health	• Define some behavior that promotes good health (eating a balanced diet, aerobic exercise, regular dental cleaning, avoid polluted places, enough sleep, ...).
Fourth year	COVID-19 transmission and ways to prevent it	• Recognize common symptoms and signs of COVID-19. • Determine ways to transmit COVID-19. • Identify ways to prevent coronavirus. • Describe everyday behaviors that promotes good health.
Fifth year	Urinary tract Nutrition and health	• Describe symptoms of a urinary tract infection. • Determine the needs of pregnant women, nursing mothers, infants, children and adolescents, and old age.
Sixth year	Nutrition and health	• Deduce symptoms of diseases resulting from malnutrition. • Mention the damage of food waste, and set out procedure for preserving and preventing it from damage.

Middle school is a transitional stage between primary and high school, it lasts three years and receives students who have a primary education certificate. In the subject of life and earth sciences, infectious diseases are taught in the third year of middle school through the unit called "Health education". The content of this unit based on the curriculum of middle school (Royaume du Maroc, 2009) is presented in Table 2.

Table 2: The content of the health education unit for the third year of middle school.

Level	Subject	Content
Third year	Body health Immunology Immune system disorders	• Protection of the body's organs (nervous system, digestive system, respiratory system, reproductive system). • Bacteria. • Specific and natural immunity. • Allergies, AIDS.

High school is an intermediate stage between middle school and university education. According to Moroccan education system of 2007 (Royaume du Maroc, 2007), teaching life and earth sciences differs in high school according to the following levels and options: the common core (first year of high school), and baccalaureate cycle in various branches and tracks (Experimental Sciences, Mathematical Sciences, Arts and Humanities). Table 3 summarizes the contents of the units related to infectious diseases for such options and levels.

Table 3 : The contents of the units that deal with the infectious diseases in high school.

Level	Subject	Content
Common core of authentic education, letters and humanities	Health and environment	Fight against epidemics (Pathogenic microorganisms, prevention and treatment).
First year of authentic education, arts and humanities	Human reproduction	Sexually transmitted diseases.
Second year of life and earth sciences	Immunology	Some immune system disorders: Acquired Immunodeficiency syndrome (AIDS).

By analyzing curricula and courses from primary to high school, we note that diseases infectious occupy an important place in these curricula. For example, COVID-19 caused by SARS-COV-2 was taught at the primary level and AIDS which is caused by HIV at middle and high school.

3.2 *Modeling the dynamics of HBV transmission through learning activities*

In this subsection, we present some modeling activities in order to teach and learn the mathematical modeling of HBV transmission.

Activity 1

When HBV spreads in a country or region, it generally causes three classes: susceptible individuals labeled by S who are not infected but have the potential getting infected, infected individuals labeled by I who are infected and able to transmit HBV, and recovered individuals labeled by R who are cured and have gained permanent immunity.

Figure 1: The flowchart representing.

1. If one day you are susceptible to be in direct contact with the blood, semen or other bodily fluids of a person infected with HBV, then what will be your health state in the following days? Add an arrow in Figure 1 expressing your answer.
2. Actually, we assume that you are infected by HBV. Express the possible cases of your state after HBV infection by adding arrows in Figure 1.
3. It is known that everyone is subject to natural death. Express this sentence with arrows in Figure 1.
4. Newborns increase the number of susceptible individuals. Express this sentence with an arrow in Figure 1.
5. Write on every arrow added in Figure 1 via the questions above, the suitable word from the following:
   - Birth rate;
   - Transmission;
   - Natural death rate;
   - Recovery rate;
   - Death rate due to HBV.
6. What does the flowchart in Figure 1 represent? Then complete the title of such figure.

The main objective of activity 1 is to help the student to construct the schematic diagram of HBV transmission illustrated in Figure 2.

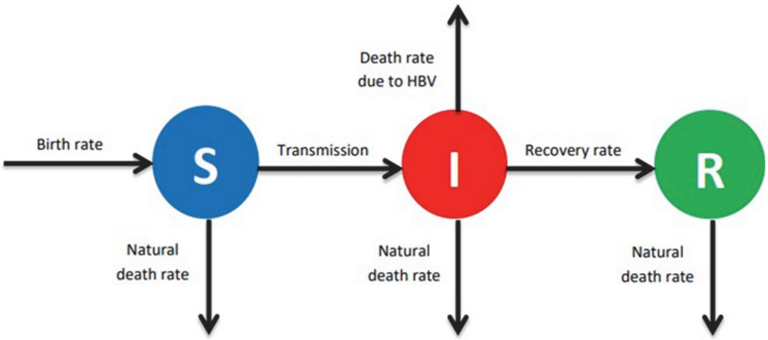

Figure 2: The flowchart representing the HBV transmission.

Activity 2

Let S(t) be the number of susceptible individuals at time t and h be a short period. In the absence of disease, we assume that such number increases through newborns and decreases due to natural death.

1. Write S(t+h) by a sentence of words. Such number, depends on what?
2. Write the relationship between S(t+h), S(t), newborns and number of new deaths during the short period h by "word equation".
3. Assume that 0.1% of the number of susceptible individuals at time t die naturally. What will be the number of new deaths at the same time t?
4. Let μ be the natural death rate. What does μS(t) represent? Assume that the susceptible individuals increase via newborns at a constant rate A. What is the biological significance of this rate?
5. Deduce that the word equation obtained in question 2 can be written in the form of the following mathematical equation: $S(t+h) = S(t) + \big(A - \mu S(t)\big)h$.

6. Calculate $\lim_{h\to 0}\frac{S(t+h)-S(t)}{h}$ and deduce that $S'(t) = A - \mu S(t)$.
7. Let $t_n = nh$, with $n\epsilon\mathbb{N}$ and $S_n = S(t_n)$. Prove that $S_{n+1} = (1-\mu h)S_n + Ah$.

Activity 2 can be used to introduce and give meaning to the notion of derivative. It can further be employed as an activity to introduce the linear ODE of the type x′(t) = ax(t) + b. Such equation was programmed in the mathematics education of the second year of baccalaureate experimental sciences, option life and earth sciences. Moreover, the last question can permit to introduce the notion of recurrent sequences of the form: $U_{n+1} = aU_n + b$ for learners by means of population dynamics systems.

Activity 3

In presence of HBV disease, the total population at time t is denoted by T(t) and it is divided as in activity 1 into three classes: S(t), I(t) and R(t) which denote the number of susceptible, infected, and recovered individuals at time t, respectively. Here, we assume that the number of susceptible individuals increases through newborns and decreases due to natural death and infection with HBV. For more details, the flowchart for the transmission of HBV is presented in Figure 2.

1. Check that $T(t) = S(t) + I(t) + R(t)$.
2. Write S(t+h) by a sentence of words. This number depends on what?
3. Justify that S(t+h) can be written as follows

S(t+h)=S(t)+(A-μS(t))h-Number of new infected cases ×h.

4. Let P_1, P_2 and P_3 be the three populations such that P_1 has a high transmission rate, several susceptible individuals and no infected individuals, P_2 has a high transmission rate, no susceptible individuals and many infected individuals, and P_3 has a zero transmission rate, several susceptible and infected individuals. How many new infected cases would occur in each population P_i?
5. Let β be the transmission rate of HBV. Choose the correct response from the following:
   The number of new infected cases at time t is equal to $\beta + I(t) + S(t)$.
   The number of new infected cases at time t is equal to $\beta I(t)S(t)$.
   The number of new infected cases at time t is equal to $\beta + I(t) - S(t)$.
   Justify your choose and deduce that
   $S(t+\mathrm{h}) = S(t) + (A - \mu S(t) - \beta S(t)I(t))\mathrm{h}$.
6. Assume that 0.2% of the number of infected individuals at time t die due to HBV and 20% of them recover. What will be the number of new deaths due to HBV and the number of new recoveries at the same time t?
7. Let d the death rate due HBV and r be the recovery rate. What does dI(t) and rI(t) represent?
8. Redo the flowchart of the HBV transmission presented in Activity 1 by using the symbols: A, βSI, μ, d and r instead of words. Prove that

I(t+h)=I(t)+[βS(t)I(t)-(μ+d+r)I(t)]h and R(t+h)=R(t)+(rI(t)-μR(t))h.

9. Let $t_n = n\mathrm{h}$ and $X_n = X(t_n)$ for all X ∈ {S, I, R}. Prove that

$$\begin{cases} S_{n+1} = (1-\mu h - \beta h I_n)S_n + Ah, \\ I_{n+1} = [1 - (\mu + d + r)h + \beta h S_n]I_n, \\ R_{n+1} = (1-\mu h)R_n + rhI_n. \end{cases} \quad (1)$$

The main aim of Activity 3 is to lead the student to construct the discrete epidemic model of HBV transmission presented by system (1) which is formulated by three difference equations.

Activity 4

1. Based on questions 5 and 8 of Activity 3, calculate
   $\lim_{h\to 0}\frac{S(t+h)-S(t)}{h}$, $\lim_{h\to 0}\frac{I(t+h)-I(t)}{h}$ *and* $\lim_{h\to 0}\frac{R(t+h)-R(t)}{h}$.
2. Deduce that

$$\begin{cases} S'(t) = A - \mu S(t) - \beta S(t)I(t), \\ I'(t) = \beta S(t)I(t) - (\mu + d + r)I(t), \\ R'(t) = rI(t) - \mu R(t). \end{cases} \quad (2)$$

3. In absence of disease, prove that
   $S(t) = \left(S(0) - \frac{A}{\mu}\right)e^{-\mu t} + \frac{A}{\mu}$ and $\lim_{t\to+\infty} S(t) = \frac{A}{\mu}$.
4. In presence of HBV, the number of susceptible individuals at the start of the infection is relatively constant and can be approximated by $\frac{A}{\mu}$. According to the second equation of system (2), verify that
   $I'(t) = (\mu + d + r)(R_0 - 1)I(t)$, where $R_0 = \frac{\beta A}{\mu(\mu+d+r)}$.
5. Establish the variation of the function I(t) in each of the following cases:
   $R_0 < 1$ and $R_0 > 1$.
6. Interpret the results obtained in question 5.
7. Give the biological meaning of the threshold parameter R_0.

In this activity, student will determine the continuous version of HBV epidemic model as well as the basic reproduction number of HBV transmission.

Activity 5

1. Enter the initial values S_0, I_0 and R_0 at $t_0 = 0$. For example, choose $S_0 = 5$, $I_0=3$ and $R_0 = 0$.
2. Based on the Geogebra slider, enter the model parameter values A, μ, β, d, r and further the time interval $[t_0, t_f]$. For example, choose A, μ, β, d, r ∈ [0, 1] and t_f varies from 0 to 100 days.
3. Enter the following commands in the input field at the bottom of the screen, taking care to type -enter- at the end of each line:
   $S'(t, I, S, R) = A - \mu S - \beta SI$
   $I'(t, I, S, R) = \beta SI - (\mu + d + r)I$
   $R'(t, I, S, R) = rI - \mu R$
4. To solve the system, enter in the input field the following command:
   NResolEquaDiff({S', I', R'}, t_0, {S_0, I_0,R_0}, t_f).
5. Calculate the basic reproduction number $R_0 = \frac{\beta A}{\mu(\mu+d+r)}$ and plot two figures one for $R_0 < 1$ and the other for $R_0 > 1$.
6. Interpret mathematically and biologically the results obtained for each figure.

In this activity, student will simulate the dynamical behavior of the continuous model (2) in the case $R_0 < 1$ and $R_0 > 1$ by using Geogebra (see Figures 3 and 4). According to Duval (Duval, 1995), this software promotes the interactions between different registers of semiotic representation. Additionally, it is familiar to students since it is downloadable free on the internet.

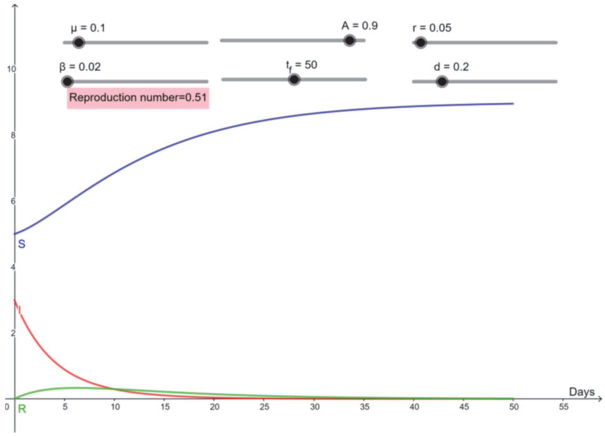

Figure 3: Dynamics of HBV transmission when $R_0 < 1$.

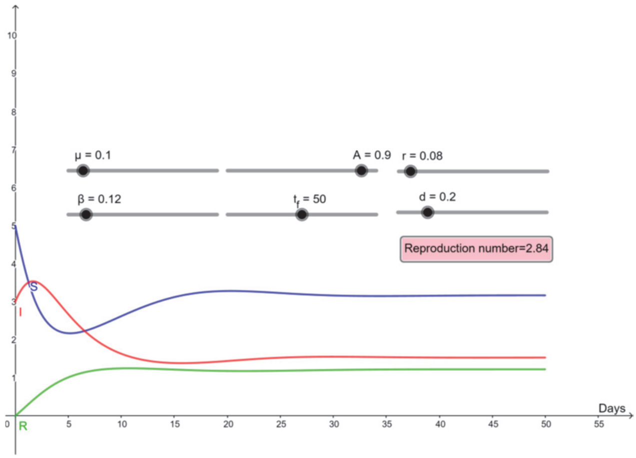

Figure 4: Dynamics of HBV transmission when $R_0 > 1$.

4. Conclusion

In this paper, we have analyzed the life and earth sciences curricula and courses from primary to high school. It is to highlight that diseases infectious occupy an important place. For example, COVID-19 caused by SARS-COV-2 was taught at the primary level and AIDS which is caused by HIV at middle and high school. We have also developed five school activities to introduce students of the second year of the baccalaureate of life and earth sciences option to the use of mathematical modeling in life and earth sciences and especially epidemiological models. We worked on the mathematical modeling of the HBV infectious disease transmission. The first activity helps student to build the schematic diagram of HBV transmission. The second activity can be used to introduce and give meaning to the notion of derivative. It can further be employed to introduce the linear ODE of the form $x'(t) = ax(t) + b$. Such equation was programmed in the mathematics education of the second year of baccalaureate experimental sciences, option life and earth sciences. Moreover, the last question of this activity can permit to introduce the notion of recurrent sequences of the form $u_{n+1} = au_n + b$ for learners by means of population dynamics systems. The third activity helps student to formulate the discrete epidemic model for HBV transmission. In the fourth activity, student will determine the continuous version of HBV epidemic model as well as the basic reproduction number of HBV transmission. The fifth activity helps students to illustrate the dynamical behavior of the continuous model describing HBV transmission using Geogebra software. We designed these activities in order to connect mathematics with life and earth sciences. In addition, these activities will contribute to development of many abilities and skills in students such as mathematical reasoning, complex problem solving, modelling and communication.

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