Research Article

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The Impact of GeoGebra on Algebraic Modeling Problem-Solving in Moroccan Middle School Students

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Abstract

The purpose of this research is to study the effectiveness of the process of modeling with GeoGebra in the development of algebraic modeling skills and to measure the impact of this program on students' interest in solving extra-mathematical and intra-mathematical problems. To achieve these objectives, we used the quasi-experimental design, where the research sample is composed of 56 students (aged 13–14 years old). The methodology consists of comparing the modeling process between an experimental group using the GeoGebra environment and a control group in the traditional environment. The results show a statistically significant difference in the students' average scores between the two groups. Generally, the experimental group that adopted GeoGebra scored higher than the other group. Interest in solving algebraic problems was higher for the experimental group. On the contrary, for the control group, students' interest in extra-mathematical problems was lower than their interest in intra-mathematical problems. On the other hand, there was no difference in the experimental group.

Keywords: *Algebraic modeling, GeoGebra, algebraic thinking, Extra-mathematical problem, Intra-mathematical problem*

1. Introduction

Teaching and learning mathematics develop mathematical skills that can be used in solving real-life challenges (Tambychik et al., 2010). Algebra is one of the most essential areas covered in high school mathematics. Indeed, much research recommends that all students master algebra, as they often use it to solve everyday problems (National Council of Teachers of Mathematics, 2000).

In addition, algebra is seen as a way to generalize, learn how to manipulate symbols, solve equations, and comprehend functional concepts. Moreover, it is a way to create a model based on real-world conditions (Stacey & Chick, 2004).

Algebra can be seen as an important subject by emphasizing that algebraic thinking is one of the most fundamental types of mathematical thinking, as it is based on skills that learners can use in many areas of life (National Council of Teachers of Mathematics, 2000; National Research Council (US) Committee on the Assessment of 21st Century Skills, 2011).

The development of algebraic thinking contributes to a good command of algebra (Kriegler, 2007). Kriegler (2007) shows that algebra can be treated as a tool for learning functions and as a way to learn mathematical modeling. According to this researcher, modeling represents one of the pillars of algebraic thinking. Squali (2000) considers the ability to construct, interpret, and validate algebraic models of real or mathematical situations as one of the main characteristics of algebraic thinking. This perspective on teaching-learning algebra allows the student to conceive the applications and relevance of algebra in everyday life (Herbert & Brown, 1997). The ability to perform mathematical modeling is considered a competency in itself that includes the skills and abilities to follow the modeling process appropriately. This process is goal-oriented and is characterized by being ready to be exploited. English (2012) describes mathematical modeling as an important strategy for solving mathematical and real-life problems. She also emphasizes the importance of learners creating models and not just using existing ones. It is an activity based on matching a real-life problem to its mathematical form through the use of representations and embodiments.

Modeling represents a bridge through which the learner can facilitate learning mathematics and make it meaningful. Indeed, it allows the representation of mathematical concepts in the form of a drawing or an embodiment; this representation allows students to relate these mathematical concepts to their reality and everyday life. It also contributes to the development of thinking and creativity. Since mathematical modeling and its applications, as well as the skills it requires, have become necessary for learners to bring something new to their learning of mathematics, it has become an element of creativity (Hansson, 2010).

In 2006, along with the competency-based approach, the guidelines for teaching and learning mathematics in Morocco were renewed. Among their recommendations, students should master certain fundamental skills, including the ability to solve real problems using the mathematical concepts they have learned (Men, 2007) and the affine functions that are the subject of this article. In addition, Kaput (2000) considers functions and quantitative relationships as languages for modeling mathematical situations or phenomena. Based on the curriculum requirements and the official pedagogical guidelines of teaching mathematics in Morocco, it is clear that mathematical modeling is a skill that secondary school students must have to reach the criteria that deem them competent.

Moroccan students have a poor impression of mathematics and generally do not perform well on mathematics tests, which are related to their ability to solve real-world problems (Mullis et al., 2012; OECD, 2019). Indeed, Moroccan students are in the gutter in mathematics and science (Mullis et al., 2020). TIMSS studies show that primary school students' performance in mathematics remains below the average score, set at 500 points (Mullis et al., 2020). Secondary school students are also at the bottom of the ranking, with 388 points out of 500 (Mullis et al., 2020).

In light of the continuous improvement of mathematics teaching and learning, educational guidelines have mentioned the importance of using technological tools to solve mathematical problems (Men, 2007). In addition to recent trends that have emerged and continued, such as (STEM) programs, Olympiad tests (TIMSS, PISA), and other tests, the student must be able to solve a large number of activities and problems related to reality.

GeoGebra is one of those technological tools that offer the possibility for students to work on more abstract structures rather than using traditional means (Faruk Tutkun & Ozturk, 2013).

GeoGebra is both a computer algebra system (CAS) and a dynamic geometry software (DGS), which makes it extremely useful in the school curriculum (Hohenwarter & Fuchs, 2004). Researchers consider mathematical modeling of real-world problems using CAS and DGS to be a problem-solving activity consistent with the goals of learning mathematics (Aktümen et al., 2013). Thus, in this study, we will attempt to answer the following questions: Does modeling with GeoGebra develop algebraic modeling skills in third-grade students? Does it generate students' interest in solving algebraic problems? Is GeoGebra able to influence students' interest in solving extra-mathematical and intra-mathematical problems?

Several studies have addressed modeling using new technologies (Greefrath et al., 2018; Latifi et al., 2022). As for algebraic thinking, most previous studies have addressed the topic through generalization (Ennassiri et al., 2022; Squalli, 2021; Vlassis et al., 2019) or problem-solving (Abouhanifa, 2021; Adihou, 2020; Moukhliss et al., 2022) by characterizing analycity in students' reasoning. However, studies that discuss algebraic thinking through modeling remain rare. It is also noted that no study has addressed the effect of GeoGebra on students' interest in solving algebraic problems, whether or not they have a connection to reality. So, this topic will be treated for the first time in this article.

2. Theoretical Framework

2.1 Modeling process and modeling skills

Essential mathematical skills can be developed in students when teachers adopt modeling and problem-solving in mathematics education (Arseven, 2015). It is important to develop real-life modeling and problem-solving skills. (Mullis et al., 2012; OCDE, 2019; Mullis et al., 2020). According to Dorier et al. (2013), mathematical modeling is a process that connects two systems by taking into account the objects involved, the relationships that connect them, and the questions asked. In this process, as well as to the construction of the model, there is a discussion and study of the correspondence between the system and its model, focusing on the dynamics and feedback of the process. According to Chevallard (1989), modeling is the schematization of a mathematical or non-mathematical system and a mathematical model of this system. Several modeling schemes can be distinguished (Chevalard, 1989; Ferri, 2006; Rodriguez, 2007). In our work, we have adopted the following scheme (Figure 1), whose approach consists in applying a known model to a real situation (Blum & Leiß, 2007).

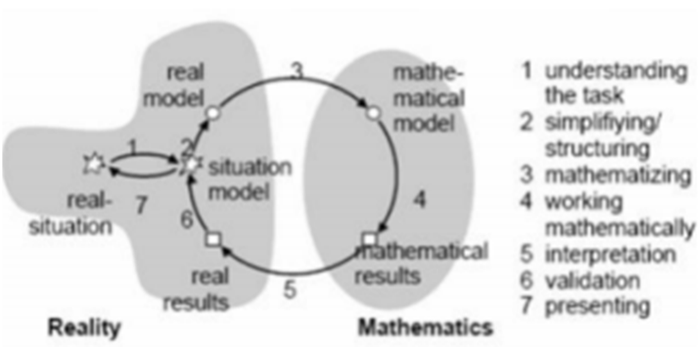


Figure 1: The modeling cycle according to Blum & Leiss (2007).

Step 1: Transitioning from the initial (real) situation to the model situation: In this step, we seek to simplify the situation, extract relevant information, and propose hypotheses for the problem.

Step 2: Transitioning from the "model situation" to the "real model": identifying the variables affecting the real situation, naming these variables, and determining their relationship.

Step 3: Moving from the "real model" to the mathematical model: we make a mathematical representation of the relevant variables and their relationships. And we simplify them, reducing their number and complexity to represent the situation.

Step 4: Working in the Mathematical Model: In this step, we use heuristic strategies and mathematical knowledge to solve the problem.

Step 5: Interpreting the mathematical results in the situation model

Step 6: Validating the results in the situation model (in case of invalidation, we have to go back to Step 2).

Step 7: Discussing the Solutions in the Real Situation.

Algebraic modeling, which is a pillar of algebraic thinking, is treated as a process that starts with the real situation, then identifies the unknowns, variables, and parameters that can be symbolized until arriving at the algebraic model, i.e., the algebraic expressions, equations, inequations, or functional relations (proportionalities or others). These constituents are constructed based on mathematical and para-mathematical competencies while passing by the "model situation."; Then, one carries out an algebraic treatment. And subsequently, an interpretation allows the validation of the solutions in the model of the situation. The operationalization of these objects is based on specific techniques in algebra (Ben Nejma, 2021). In our study, we have focused on modeling through functional relations and specifically on modelling, using affine functions.

2.2 Reorganizing the modeling process with Technology

There is an obvious relationship between technological tools and mathematical modeling (Greefrath et al., 2018). This explains the multiplicity of models that include technological tools in a modeling cycle (Blum & Leiß, 2007; Siller & Greefrath, 2010). In the modeling process, students are invited to generate discussions based on the mathematical models they have built using digital technologies; in other cases, they work with technological tools from the beginning of the modeling process to its end. Technology can be considered a means to reorganize the modeling process (Borba & Villarreal, 2005; Diniz & Borba, 2012). In this sense, technology responds to the needs that arise during the modeling process: accessing and analyzing data, creating simulations of the studied phenomenon, comparing models and results, validating or publishing a model, etc. Therefore, technology, in this case, is a tool that reorganizes the modeling process, allowing students to solve complex mathematical problems where the latter should not present a constraint to their studies (Molina-Toro et al., 2019).

Three groups of tools can be used in the modeling process: Computational Algebra Systems (CAS), Dynamic Geometry Software (DGS), and Spreadsheets (SP) (Barzel et al. 2005). Computational Algebra Systems (CAS) can be used to perform certain operations, parameterize data, and analyze graphs (Possani et al., 2010; Rodríguez & Quiroz, 2016; Trigueros, 2009). Concerning DGS, it allows us to study different mathematical concepts through different semiotic registers (animations, tables) and to verify the models developed by the learners (Sekulić & Takači, 2013; G. Stillman, 2011). For SP, it allows us to record, organize, and analyze data. For example, collect and organize data to create scatter and regression plots and find an appropriate model to solve certain mathematical problems (G. Stillman & Brown, 2014) or representations to discuss, interpret, or validate models (Daher & Shahbari, 2015). Choosing the right tools, such as GeoGebra, can enhance this support.

GeoGebra is a tool that plays several roles. It allows us to experiment or explore (Fahlgren & Brunström, 2014). Thus, we can draw and build using it (Hohenwarter et al., 2008). GeoGebra also has a role in calculating results (numerical, algebraic), which can save time (Siller & Greefrath, 2010). In addition, GeoGebra allows for self-checking of results and is used to make presentations. These digital tools help students understand modeling contexts, which has a significant impact on mathematical modeling (Carreira et al., 2013.; Rodríguez Gallegos & Quiroz Rivera, 2015).

2.3 Teaching mathematical modeling with GeoGebra (an example)

We will present an example of an activity (Hall et Lingefjärd, 2016) that illustrates the use of the GeoGebra program in algebraic modeling. The example below was presented to the students during the experimentation phase.

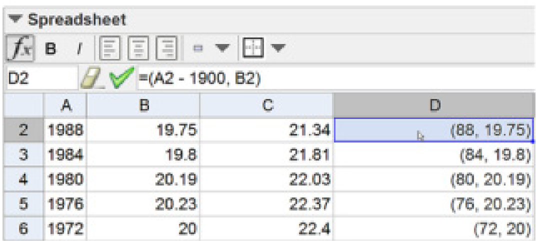
Example: Based on the Olympic gold medalists in the 200-meter race, mathematical models can be constructed to predict and compare the women's and men's 200-meter records at future Olympic Games and World Championships. Table 1 shows the results of the Olympic gold medalists in the 200 meters.

One must look for a mathematical model that describes the behaviour of the phenomenon to be studied so that predictions can then be made about Y (called the dependent variable) when X (called the independent variable) is measured. In this work, we will be interested in solving this problem with the help of the GeoGebra tool, which will facilitate this task. What are the steps to follow to find this model?

Year	Men	Time (s)	Women	Time (s)
1988	J. DeLoach, USA	19.75	F. Griffith-Joyner, USA	21.34
1984	C. Lewis, USA	19.80	V. Brisco-Hooks, USA	21.81
1980	P. Mennea, Italy	20.19	B. Wöckel, East Germany	22.03
1976	D. Quarrie, Jamaica	20.23	B. Eckert, East Germany	22.37
1972	V. Borzov, Soweet	20.00	R. Stecher, East Germany	22.40
1968	T. Smith, USA	19.83	I. Szewińska, Poland	22.5
1964	H. Carr, USA	20.3	E. McGuire, USA	23.0
1960	L. Berruti, Italy	20.5	W. Rudolph, USA	24.0
1956	B. Morrow, USA	20.6	B. Cuthbert, Australia	23.4
1952	A. Stanfield, USA	20.7	M. Jackson, Australia	23.7
1948	M. Patton, USA	21.1	F. Blankers-Koen, Holland	24.4
1936	J. Owens, USA	20.7		
1932	E. Tolan, USA	21.1		
1928	P. Williams, USA	21.8		
1924	J. Scholtz, USA	21.6		
1920	A. Woodring, USA	22.0		
1912	R. Craig, USA	21.7		
1908	R. Kerr, Canada	22.6		
1904	A. Hahn, USA	21.6		
1900	W. Tewksbury, USA	22.2		

Figure2: Olympic Winners in the 200 Meter Sprint

- Create a table containing the results obtained after 1988.
- Introduce the table entries in the spreadsheet, which can be found in the menu View>Spreadsheet, or by pressing Ctrl-Shift-S directly.
- In column A, enter the information for the male runners and type (A2-1900, C2) in cell D2. This gives us a point in the graph that represents men's records in 1988.
- Generate the other points that concern the men's results by dragging the fill handle downward to generate the rest of the points after selecting cell D2. We will have the table shown in the figure.



	A	B	C	D
2	1988	19.75	21.34	(88, 19.75)
3	1984	19.8	21.81	(84, 19.8)
4	1980	20.19	22.03	(80, 20.19)
5	1976	20.23	22.37	(76, 20.23)
6	1972	20	22.4	(72, 20)

Figure 3: Information stored in the GeoGebra spreadsheet

- Then we seek to perform a linear regression on these points using the following instructions: Select the points in the spreadsheet, right-click on them, and select Create…>List, then enter the Fit Line[list1] command in the input field.
- Repeat the same instructions to display the women's data.
- With the intersect tool, click on the graphs to find the intersection between them, as shown in the figure.

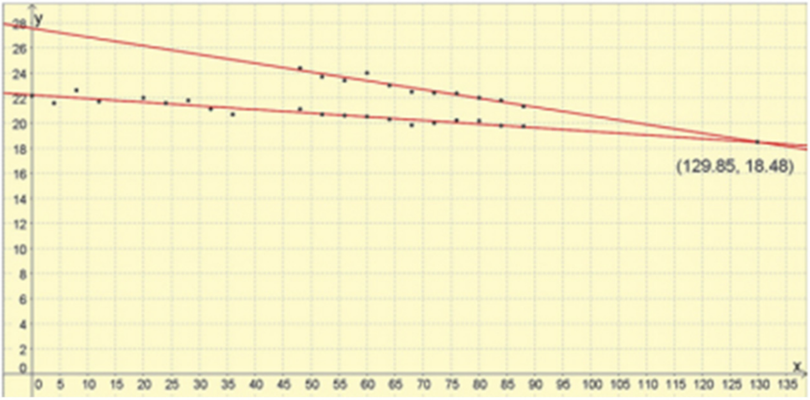


Figure 4: Women's record times as seen to decrease faster than men's.

Women will beat men by about 82 years after 1948. Around 2030, the Olympic 200-meter record times for men and women will have fallen to 18.48 seconds.

2.4 Extra-mathematical and intra-mathematical problems

Mathematical problems can be divided into two types: extra-mathematical problems, which have a link with reality (real-world problems), and intra-mathematical problems, where the link with reality is not established.

The difference between these two types of problems lies in the cognitive processes required to solve them (Blum & Leiß, 2007; Galbraith & Stillman, 2000; Verschaffel et al., 2000). Extra-mathematical problems (see example 1) are reality-based, involve real objects such as those in nature and everyday life, and are characterized by complex cognitive processes (Schukajlow et al., 2012; Niss et al., 2007).

Example 1: Extra-mathematical problem

The price paid for a cab trip includes a fixed charge and a sum proportional to the number of kilometers traveled.

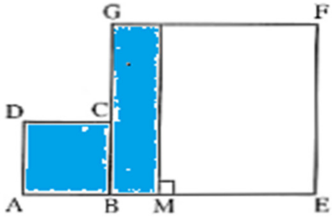
A passenger named P1 paid 12 DH for a 4-kilometer trip, while P2 paid 18.25 DH for a 6.5-kilometer trip. Determine the amount of the pickup and the price per kilometer traveled.

In the first step of the solution, students create a model of the situation to understand the problem. Then, they mathematize the problem by looking for an appropriate mathematical model where they perform mathematical operations to find a mathematical result, and finally, they interpret and validate the solution concerning reality (Blum & Leiß, 2007; Galbraith & Stillman, 2006; Schukajlow et al., 2012; Verschaffel et al., 2000). Whereas intra-mathematical problems (see example 2) are based on objects and relations belonging to the domain of mathematics (e.g., a geometrical problem) and do not require moving from the real world to the mathematical world…

Example 2: Intra-mathematical problem

In the given figure, ABCD and EFGH two squares of sides 5 and 6, respectively; M is a point on the side [BE], and we pose AM=x

1. Determine the area of the blue section for x = 5,5 and x = 7.
2. Express the area f(x) of the blue part as a function of x.
3. Determine the position of point M so that the area of the square BEFG is twice the area f(x).



The mathematical model is presented directly. To find the result, students can apply mathematical procedures immediately; this result does not have to be interpreted in relation to the real world.

In learning mathematics, both types of problems (intra-mathematical and extra-mathematical) are essential, as they help students have a concrete understanding of mathematical ideas and lead them to acquire the ability to relate mathematical knowledge to the real world and also to practice procedures and techniques of a mathematical nature (Beswick, 2011; Schukajlow et al., 2012).

3. Materials and Methods

3.1 Research design

This study aims at testing the effectiveness of the dynamic mathematics software GeoGebra in learning mathematical modeling. To answer the research questions, we used a quasi-experimental approach based on two of the groups (experimental and control) to know the impact of the independent variable (modeling with GeoGebra) on the dependent variables, which are the algebraic modeling and the interest of the students in solving intra- and extra-mathematical problems.

3.2 Research objectives

The purpose of this study is to:

1. Determine the effect of GeoGebra software on the process of algebraic modeling in third-year middle school students while studying affine functions:
2. Investigate the effect of modeling with GeoGebra on students' interest in solving algebraic problems
3. Raise the difference in students' interest in solving intra-mathematical and extra-mathematical problems
4. Investigate the effect of modeling with GeoGebra on students' interest in solving both extra-mathematical and intra-mathematical problems.

3.3 Sample and data collection

The study group included 3rd-grade middle school students (13–14 years old) in Casablanca (Morocco) during the 2021–2022 school year. With a total of 56 students—27 in the experimental group and 29 in the control group—the experimental group received GeoGebra-based instruction using laptops connected to WIFI networks, while the control group received traditional constructivist method.

Several GeoGebra activities were devoted to this study, including the previous activity (see the example of women running faster in this article).

Both groups were given a pre-test to ensure equivalence before beginning the study. The study lasted two weeks, with both groups taking a post-test at the end of the fourth week.

3.4 Data Collection Tools

3.4.1 A Scoring grid Using Common Core State Standards for Mathematics (CCSSM) for Modeling Cycle (Leong, 2012)

To measure the students' modeling process, we used a rubric (see table) using the Common Core State Standards for Mathematics (CCSSM) for the modeling cycle, developed by Leong (2012). It was developed based on the important checklist that is required in the modeling process. The scores are based on 5 points ranging from 0 to 4. 0: Not done 1: Below acceptable 2: fair; 3: good 4: Excellent. Weights between 1 and 3 will be weighted according to the importance of each step. e.g., Formulating a model section will be weighted 3 due to its importance, i.e., the total possible points for this step will be 36 since a score from 0 to 4 must be assigned to each element of this section.

3.4.2 Measuring students' interest in problems by type (mathematical or non-mathematical)

To measure students' interest in engaging in the proposed tasks, we used a tool and scale validated in the study by Rellensmann & Schukajlow (2017). Immediately after the treatment of the intra-mathematical and extra-mathematical problems, we asked students the following question ("was it interesting to work on this problem?"), Students were asked to respond using the 5-point Likert scale to record their responses (1=not at all true, 5=very true).

3.5 Intervention

The teacher began the experiment with an exploratory session for the experimental group to become familiar with the GeoGebra program and discover its functionality. Both groups performed the pre-test on the first day of the study. Then, there were four sessions, two per week. Each session lasted two hours, for a total of six hours of lectures and exercises on affine functions and their applications in modeling. Both groups took a post-test on the last day of the study. The pre- and post-test scores of the experimental and control groups were used to generate the data for this study. The researchers used two rubrics to assess students' modeling abilities and interests in intra-mathematics and extra-mathematics problems.

3.6 Data Analysis

Independent samples t-tests were performed to determine significant differences between the two experimental and control groups in the post-test and pre-test scores. We also examined the normality and homogeneity of score variances.

4. Results

4.1 Student performance on the pre-test

We designed a pre-test to verify the equivalence of the two groups, control and experimental. This pre-test assesses the students' prerequisites in algebra: operations on numbers, development and factoring, equations, proportionality, and linear functions.

Students in the control group scored a mean of 10.00 (S.D = 2.69), while the mean was 9.52 (S.D

= 2.37) for the experimental group. The t-test revealed no significant difference (t = 0.706; P = 0.483), the two groups were equivalent in their algebra prerequisites.

Table 1: Students' Performance in Pre-test

	Experimental Group		Control Group		t	p
	Mean	S.D	Mean	S.D		
Pre-test	9.52	2.37	10.00	2.69	0.706	0.483

4.2 Assessing modeling tasks

Table 2: Modeling Cycle Scoring Rubric

Process	Experimental Group (N = 27)		Control Group (N = 29)		t	p
	Mean	S.D	Mean	S.D		
Identifying Variables (12 Pts)	7.26	2.94	7.72	2.49	0.64	0.525
Formulating a Model (36 Pts)	19.41	3.77	17.90	4.80	1.30	0.199
Mathematical Operations (24 Pts)	12.19	2.94	12.79	3.39	0.71	0.479
Interpreting the Results (36 Pts)	17.56	6.69	12.83	6.62	2.65	0.01
Validating the Conclusion (24 Pts)	17.04	4.89	10.69	2.60	5.99	P < 0.001
Reporting on Conclusions (8 Pts)	4.22	2.67	1.97	2.14	3.49	0.001

After the end of our experiment, both groups took a post-test. Students' skills in the algebraic modeling process were measured. The first three steps, identification of variables, formulation of the model, and mathematical operations did not differ between the experimental and control groups. The scores were similar between the two groups.

The last three steps showed significant differences in favor of the experimental group. Indeed, in the stage of interpretation of the results, the experimental group obtained 17.56 (S.D. = 6.69) while the control group obtained 12.83 (S.D. = 6.62), and the t-test revealed a significant difference (t = 2.65; p = 0.01). In the stage of validation of the results, the experimental group obtained 17.04 (S.D. = 4.89) and the control group obtained 10.69 (S.D. = 2.60), and the t-test revealed a significant difference (t = 5.99; p < 0.001). In the stage of reporting the conclusions, the experimental group obtained 4.22 (S.D. = 2.67) and the control group obtained 1.97 (S.D. = 2.14); the t-test revealed a significant difference (t = 3.49; p = 0.001).

Table 3: The total score of the modeling competence

Domain	Experimental Group		Control Group		t	p
	Mean	S.D	Mean	S.D		
Problem	77,67	10,749	63,90	9,053	5,198	P < 0.001

We compared the total scores of the two groups. Students in the experimental group performed better, scoring 77.67 (S.D = 10.74). Students in the control group scored a mean of 63.90 (S.D = 9.05). The difference is significant (t = 5.19; p < 0.001).

4.3 Assessment of students' interest in extra-mathematical and intra-mathematical problem solving

Table 4: Comparison of students' interest in solving problems with and without problems with and without a link to reality

Domain	Experimental Group		Control Group		t	p
	Mean	S.D	Mean	S.D		
Extra-mathematical problem	4.07	0.78	1.90	1.01	8.96	P < 0.001
Intra-mathematical problem	4.19	0.83	3.14	0.99	4.26	P < 0.001

Table 4 shows the results of the two groups for the two types of problems. The results show a significant superiority in favor of the experimental group. In the extra-mathematical problems, the experimental group obtained 4.07 (S.D. = 0.78) while the control group obtained 1.90 (S.D. = 1.01); the t-test revealed a significant difference ($t = 8.96$; $p < 0.001$).

In the intramathematical problems, the experimental group obtained 4.19 (S.D. = 0.83) and the control group obtained 3.14 (S.D. = 0.99); the t-test revealed a significant difference ($t = 4.26$; $p < 0.001$).

Table 5: Comparison of intra-mathematical and extra-mathematical problems for each of two groups

Domain	Extra-mathematical problem		Intra-mathematical problem		t	P
	Mean	S.D	Mean	S.D		
Control Group	1.90	1.01	3.14	0.99	4.75	P < 0.001
Experimental Group	4.07	0.78	4.19	0.83	0.47	0.640

Another comparison was made to see which type of problem performed better for each group. For the control group, the students performed better on intra-mathematical problems. They scored 1.90 (S.D. = 1.01) in extra-mathematical problems, while the average was 3.14 (S.D. = 0.99) for intra-mathematical problems, with a significant difference ($t = 4.75$; $p < 0.001$).

For the experimental group, students scored similarly on intra-mathematical and extra-mathematical problem types. They scored 4.07 (S.D = 0.78) in extra-mathematical problems and 4.19 (S.D = 0.83) for intra-mathematical problems with no significant difference ($t = 0.47$; $p = 0.64$).

5. Discussion

The purpose of this study is to determine the effect of using the GeoGebra program on Moroccan middle school students' acquisition of mathematical modeling skills, and to assess their interest in solving reality and non-reality problems.

The results show that the GeoGebra program does not affect the first three steps of the modeling process: (1) identification of variables, (2) formulation of a model, and (3) operation on the model. This may be because the study was implemented in a short time, which is consistent with its results (Masri et al., 2016). The GeoGebra program improves students' performance in the long run. On the other hand, it was found that there is an impact of the GeoGebra program in the last three steps of the process: (4) interpretation of results, (5) solution validation, and (6) presentation of conclusions in favor of the experimental group. This was because teaching modeling with GeoGebra focuses more on distinguishing between the result of the solution as an arithmetic value, and the meaning of that result in the problem situation. This property translates into the learner's thinking through his distinction between the internal validity of the result and its external validity. It may be that the result is algebraically and mathematically correct, but the student does not give any importance to its meaning, concerning what is needed to solve the problem. A study (Kilpatrik, Silver, and Days, 1999) confirms the effectiveness of teaching methods that aim to develop the ability to verify the accuracy of the mathematical solution obtained. They believed that this verification aided the student in comparing the result to the solution condition, indicating the student's ability. Modeling with GeoGebra clearly improved the ability to interpret the result as the correct answer to the problem posed. The solution, even if it is mathematically correct, is considered provisional. Modeling with GeoGebra makes the ability to communicate mathematical solutions a link that gives meaning to algebraic modeling for students. Students understand that determining the mathematical solution is only the first step in determining solutions to the problem at hand. In the same sense, the results by Verschaffel et al. (2000) show that the ability to communicate solutions in written or verbal form is considered one of the skills for mathematical modeling of the problems presented.

Generally, the results of the statistical analysis of the data showed that the modeling strategy with GeoGebra has a positive effect on the development of students' algebraic modeling skills using the tools that the GeoGebra program has, which allow the student to interact directly with the educational content. He becomes the center of the educational process and can solve problems in a short time and with little effort. In this sense, Bayazit & Aksoy (2010) see that using the GeoGebra program supports structural and procedural knowledge and helps build graphical models to solve algebraic problems.

The second research question dealt with students' interest in solving mathematical problems. The results showed that there was a significant difference between the two groups. Most students in the experimental group felt from the beginning that the digital tool GeoGebra could lead them to success in the challenge that had seemed difficult at the beginning, which developed the students' self-confidence and increased their interest in solving problems. Learning to model using GeoGebra gives the student a view of mathematics different to the one they have before. Using GeoGebra makes mathematics more dynamic. It addresses minds and thoughts, not just rigid symbols or fixed models. This may be what made the experimental group interact better in the lessons of this unit and strive to improve their performance. They also feel comfortable and relaxed while learning. It can be said that using GeoGebra in learning mathematics has reduced some students' chronic hatred towards mathematics, as they have practiced it differently and unusually, which is largely in line with their daily hobbies and their interest in using technological means. These results are consistent with studies that have addressed the positive effect of technological tools such as GeoGebra on positive attitudes toward learning mathematics (Arbain & Shukor, 2015; Latifi et al., 2022; Yılmaz et al., 2010).

The other objective of this study was to answer the research question regarding the impact of GeoGebra on students' interest in solving problems with and without a link to reality. The control group showed more interest in problems that were linked to reality. These results are surprising because teachers often think students are interested in solving real-world problems (Cordova & Lepper, 1996; Mitchell, 1993). These results agree with the study by Rellensmann & Schukajlow (2017). On the other hand, the results show that there is no significant difference between the two problems for the experimental group. GeoGebra increased students' interest in extra-mathematical problems and brought them closer to their interest in intra-mathematical problems.

6. Conclusion

The results show that modeling with GeoGebra has a positive impact on the development of algebraic modeling skills. Generally, it can be said that GeoGebra can be considered a teaching material that has the potential to contribute to the development of algebraic thinking. Reconsidering the results of the study in the field of teaching mathematics, we assume that the work on the integration of the GeoGebra program in the teaching of mathematics is important for the different phases of teaching because of its effectiveness in the development of algebraic thinking skills in students of different levels. We also recommend preparing and qualifying teachers to use pedagogical programs that help to develop different types of thinking in students, including algebraic thinking, and the GeoGebra program in particular because of its effectiveness and high quality in dealing with different mathematical axes.

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