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Modeling with Differential Equations and Geogebra in High School Mathematics Education

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Abstract

The integration of technology in mathematics education is still much slower despite the benefits of using technology. Dynamic Geometry Computer Software (DGCS), like GeoGebra, makes mathematical modeling alive and more attractive. They allow dynamic graphical representations and support an in-depth understanding of abstract mathematical concepts. From this perspective, the purpose of this study is to design activities (which include the use of mathematical software Geogebra), where the students manipulate, observe, analyse, and generalize mathematical concepts. With GeoGebra, solving more interesting and difficult problems will evidently be more feasible for students. To evaluate the contribution of GeoGebra in the learning of mathematical modeling, we carried out an experimental study in two groups for the 12th grade students. Experimental group (using GeoGebra) and control group (not using GeoGebra). The intervention lasted five weeks on the subject of differential equations. Based on the results, students in the experimental group achieved higher scores on the Mathematical Modeling Skills Rating Scale.

Keywords: *Mathematical modelling; Differential equations; GeoGebra environment; Secondary school, Mathematical modelling competencies*

1. Introduction

The major goal of mathematics education is to enable students to solve mathematical problems and use mathematics in real-world contexts (EACEA, 2011; NCTM, 2000). Students must work with realistic, authentic, and messy tasks (Abassian et al., 2020). The necessity, application, or relevance of mathematics to the Science, Technology, Engineering, and Mathematics fields (STEM) or to their everyday lives is not felt by students (Kaiser et al, 2011). Therefore, secondary school students lack motivation and even question the utlity of studying mathematics (Schukajlow et al, 2017). Student International Assessments (PISA, TIMSS) use problems in real-world context, which is often referred to as 'modeling process' to assess an individual's capacity to identify and understand the role that mathematics plays in the world (OECD, 2003; Nilsen et al, 2013). When we analyze the subject of teaching mathematics, primary and secondary school students in Morocco think that mathematics is unlikeable, and it has been observed that they are unsuccessful in mathematics exams (Mullis et al, 2012; OECD, 2019). The Trends in International Mathematics and Science Study (TIMSS) results have revealed that Morroco is lagging behind as far as the students' knowledge of maths and science is concerned (Mullis et al, 2020).

In mathematics, Moroccan primary school students remain far from the average TIMSS score, set at 500 points. They are also far behind their peers from Singapore (score of 625) which tops the ranking alongside Hong Kong, South Korea, Taiwan and Japan (5th place in the world) (Mullis et al, 2020). Moroccan secondary school students are not much better. Indeed, for mathematics, Morocco is at the bottom of the ranking, with a score of 388, behind Oman (score of 411), Kuwait, Saudi Arabia, and South Africa (389) though it has improved its previous score by 4 points. It still remains far from the average by 112 points. Additionaly, it is 228 points behind Singapore, which tops the ranking again (Mullis et al, 2020).

According to the Program for International Student Assessment (PISA), version 2018, Morocco is placed 75 out of 79 countries that took part in the assessment which evaluated 15-year-old students' abilities in reading, science, and mathematics. The Moroccan students' mathematics score was 368 points, over 100 points below the international average of 489 (OECD, 2019).

In Morocco, secondary Mathematics curriculum was renewed in 2006. Since then no change has been absereved in relation to ifferential equations topics which are the concern of this research. The course of differential equations is presented without any modeling activities, or the introduction of mathematical software (MEN, 2007). These equations are crucially important in mathematical modelling. Problems are accordingly solved symbolically, analytically, or numerically (Hall and Lingefjärd, 2016). Dynamic graphical solutions can be attained via a spreadsheet like Excel or GeoGebra (Hall and Lingefjärd, 2016). Many real-life problems can be modeled using differential equations; they have an important role in a variety of domains such as Economics, Physics, Biology, etc (Burghes and Huntley, 1981; Hattaf et al, 2012; Krutikhina et al, 2018). In mathematics, a differential equation is important as it has interrelations with many mathematical concepts including functions, derivatives, integrals, etc (Arslan, 2010). Our research is based on the idea that the introduction of modeling activities with Geogebra improves students' mathematical skills and motivation during differential equations courses in high school.

2. Modeling Process and Modeling Competencies

Mathematical modeling and mathematical problem solving helps to build skills essential in mathematics education (Arseven, 2015). The development of individual skills to model and solve real-life problems is very important (Mullis et al, 2012; OECD, 2019; Mullis et al, 2020). Many studies have discussed the teaching and learning of mathematical modeling (Arseven, 2015; Boaler, 2001). Boaler (Boaler, 2001), mathematical modeling theory suggests that knowledge is created as a result of a series of interactions between people and the world. The scheme proposed by Rodriguez (2007) (Rodriguez, 2007) constitutes a design for composing a differential equation (DE):

1. Understanding the situation (problem), simplifying, structuring, and identifying the relevant quantities;

2. Choosing the correct variables and accompanying units;
3. Identifying the correct dependent variable and expressing how this variable changes in the form of a differential equation;
4. Expressing the measure of change in a DE perhaps preceded by the starting values;
5. Interpretation, verification, and communication.

Both Rasmussen and Marrongelle (Rasmussen and Marrongelle, 2006), as Rodriguez (Rodriguez, 2007), note the difficulty that students have in composing a DE. The Programme for International Students Assessment (PISA), defines mathematical literacy as the capacity of 15-year-old students to analyze, reason, and communicate effectively as they pose, solve, and interpret mathematical problems in a variety of situations (OECD, 2003). A modeling diagram summarizes the transitions that exist between the different stages of a modeling process. The word cycle or modeling process is also used. We can distinguish several modeling schemes (Ferri, 2006; Chevallard, 1989). In our work we have adopted a scheme that we find very adequate for our research because it is particularly focused on the starting situations (real situation), the physical model, and the mathematical model (Rodriguez, 2007).

This modeling scheme comprises 9 steps (Figure 1):

Step 1: Real Situation (RS): We start from a real situation and make a simplified and precise description of the phenomenon.

Step 2: Mental Situation Representation (MSR): In this step, we make assumptions using a drawing, diagram, etc.

Step 3: Pseudo-concrete model (PCM): This is an intermediate phase where the presentation of the model is made in current terms (state the laws, the unknowns, the variables for comparing the hypotheses, etc.).

Step 4: Physical Model (PM): This step is based on the use of conventions and coding specific to the physical theme studied.

Step 5: Mathematical Model (MM): In this step, we establish a set of mathematical equations or formalisms, which represent the properties of the model, and the assumptions made.

Stage 6: Mathematical Study (ME): It is a phase devoted to purely mathematical work.

Step 7: Physical Results (PR): It is the step where we translate the mathematical results in terms of physics.

Step 8: Pseudo-Concrete Results (PCR): It is a step during which we once again translate the physical results obtained in pseudo-concrete terms; that is to say, in terms that come into play during the description of the real situation.

Step9: This is the step of confronting the model with the actual starting situation.

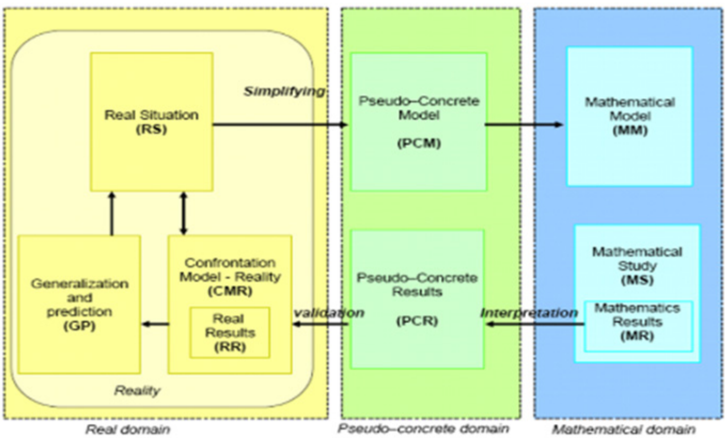

Figure 1: A modeling scheme (Rodriguez, 2007)

3. Mathematical Modeling in GeoGebra Environment

Computer algebra systems (such as Derive, Mathematica, Maple or MuPAD) and dynamic geometry software (such as Geometer's Sketchpad, Geogebra or Cabri Geometry) are powerful technological tools for teaching mathematics by encouraging discovery and experimentation in classrooms (Lavicza, 2007; Ardıç and İşleyen, 2018; Buteau et al, 2010; Stephens and Konvalina, 1999; Stols and Kriek, 2011). GeoGebra is an interactive geometry, algebra, calculus and statistics computer software which gives a chance to students, from primary school till higher education, to work on more abstract structures rather than using traditional means (Tutkun and Ozturk, 2013). At the same time, GeoGebra is characterised by being a computer algebra systems (CAS) and as dynamic geometry software (DGS), which makes it very useful in the school education program (curriculum) (Hohenwarter and Fuchs, 2004). GeoGebra can best be described as a mathematical laboratory. It handles dynamic geometry, graphing, spreadsheets, statistics, regression, algebra, matrices, complex numbers, differential equations, symbolic algebra, programming, and so on (Hall and Lingefjärd, 2016).

Mathematical modeling of real-world problems with CAS and DGS is viewed by researchers as a problem-solving activity that is consistant with the purposes of mathematical learning (Aktümen et al, 2013). In particular, mathematical modeling facilitates understanding of mathematical concepts, the learning of new mathematical concepts, and the establishment of interdisciplinary relationships by demonstrating in detail the applicability of mathematical concepts in real-life (Zbiek and Conner, 2006). In the present work, we show how GeoGebra can be used for mathematical modeling and general mathematics teaching.

4. Teaching Mathematical Modeling with Geogebra (An Example)

We will illustrate gradually how GeoGebra can solve a modelling problem. The following activity (Hall and Lingefjärd, 2016), is part of the course presented to the students of our study.

Activity: Cooling Temperatures

A small amount of water cools rapidly after being poured from a kettle into a cup. The evolution of the temperature as a function of time is presented in Table 1. The room temperature was 22.3°C.

Table 1: Cooling Temperatures

t (s)	*T* (°C)	*t* (s)	*T* (°C)	*t* (s)	*T* (°C)
0	69.58	360	43.96	720	35.00
30	66.11	390	42.92	750	34.53
60	61.41	420	41.95	780	34.04
90	58.07	450	41.05	810	33.59
120	55.60	480	40.18	840	33.20
150	53.58	510	39.40	870	32.76
180	51.66	540	38.70	900	32.37
210	50.05	570	38.00	930	32.00
240	48.52	600	37.32	960	31.64
270	47.24	630	36.67	990	31.30
300	46.00	660	36.08		
330	44.96	690	35.50		

1. We open the GeoGebra spreadsheet and enter the data. The distribution of these points is exponential and decreasing (Figure 2). We can use an exponential fit $f(x) = c.a^x$.
2. We type $T_0 = 22.3$ and then = B2-T_0 in the correspondant cell.
3. Fit an exponential function to the new data.
4. The fit may not be as good as you might have initially expected. Erase the first three points and press Enter to create the new list. Fit an exponential function to the new data.

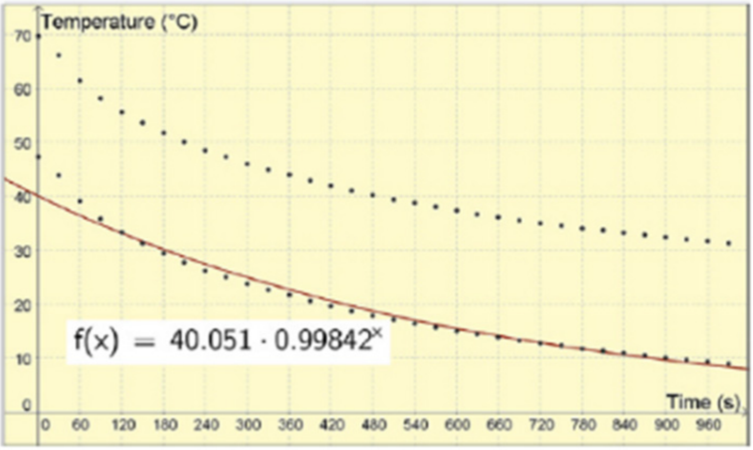

Figure 2: Exponential fit to data

5. We use this model to predict the time it will take to cool the water to 26 °C.
6. Fit a model: $m(x) = Cr^x + t_0$, where t_o is the ambient temperature to the data (Figure 3).

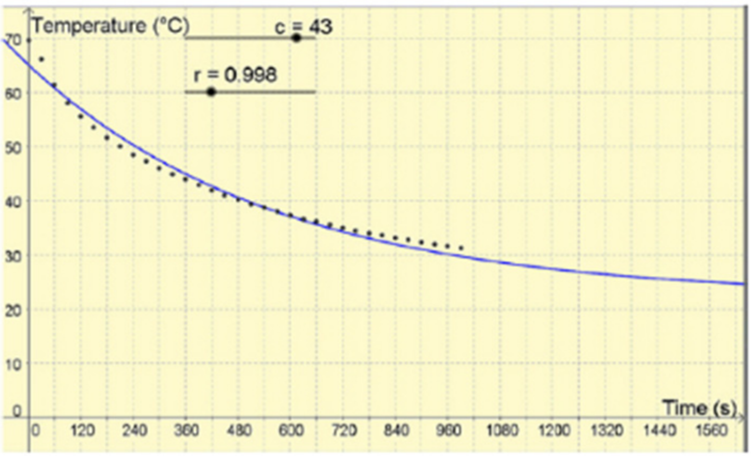

Figure 3: General fit to the data, using the model m(x)

7. Using the sliders, adjust the model function to lie roughly along the data points.
8. Construct a mathematical model that fits all these data by using Newton's cooling law: (T' = k. ($T-T_o$) =k. ΔT).
9. Newton's cooling law is not the only possible model, Try T' = k. ΔT^p, where p = 5/4 or p = 4/3 depending on the type of flow you have at the contact surface.

Type SolveODE [y'= k y^(5/4)]. The answer is a rational function: $T(t) = 256/(c-k.t)^4$

10. Try to fit the data to a model function based on: $m(x) = 256/(c-k.x)^4 + 22.3$ (Figure 4)

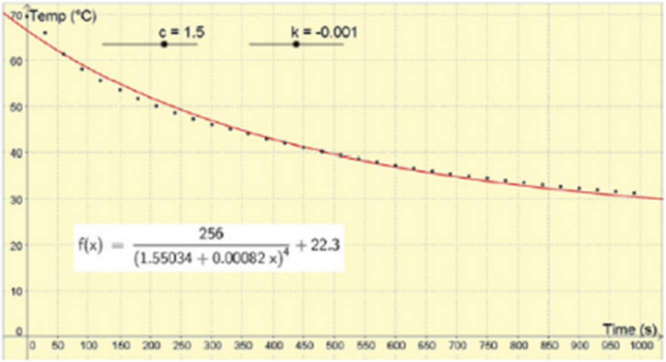

Figure 4: Fitting a rational function to the data

Neverteleless, there are still systematic errors (Figure 4). Will the exponent 4/3 have better result? In such a case, the solution for the differential equation is $m2(x) = 27/(c+kx)^3 + 22.3$ (Figure 5).

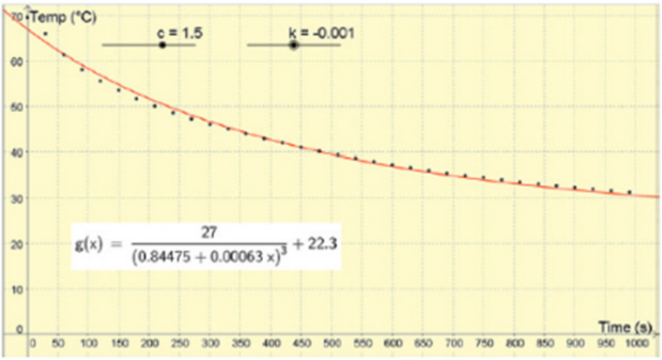

Figure 5: New attempt with better—but still not completely satisfying—results

11. We consider a general function $m3(x) = a/(c+kx)^p + 22.3$ with modifiable parameters a, c, and p. Set a = 22.4, c = 0.6, k = 0.00113, and p = 1.4 in order to get reasonable starting values (Figure 6)

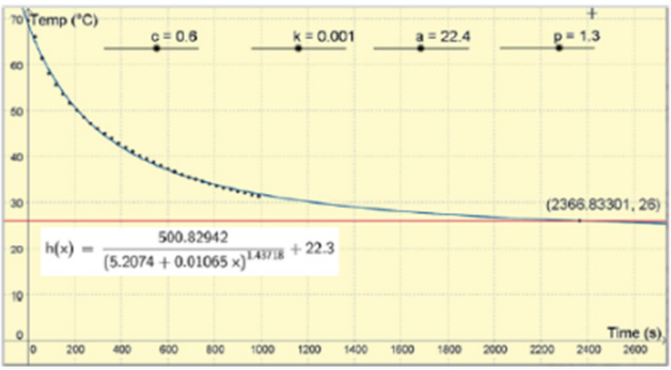

Figure 6: Rational function with an arbitrary exponent

12. This time, we get a very good fit to the data values. The downside is that the function obtained is not linked to a specific differential equation. We plot the line y = 26 in Figure 6 and find the intersection between the line and our function by typing Intersect [a, h]. We get an answer 39.5 min.
13. Around p = 2.2, the resulting function $m4(x) = 1/(c+kx)^{2.2} + 22.3$ seems to follow the data (Figure 7). In this case, the answer is about 31.5 min, close to the experimental result.

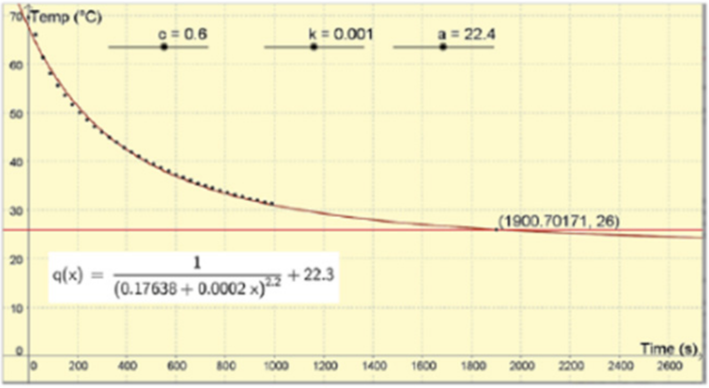

Figure 7: Best answer so far

5. Experimentation

5.1 Purpose and Importance of the Study

Mathematical modeling is a task that requires various skills, including real-world knowledge (Burkhardt, 2004; Ikeda, 2007). A question that math teachers ask themselves is "how do we assess the process of mathematical modeling?". In this context, the aim of this study is to evaluate the modeling process and the contribution of GeoGebra in the development of modeling skills for the 12th grade students. To this end, this study tries to answer the following research questions:

1. How can teaching mathematical modeling in a GeoGebra environment improve students' modeling skills?
2. How do students appreciate the introduction of GeoGebra in modeling activities?

5.2 Materials and methods

5.2.1 Research design

This study used the quasi-experimental design with two groups (experimental group and control group) to test the efficiency of Geogebra in mathematical modeling learning. The control group was taught through traditional method. The experimental group was taught using dynamic mathematics software Geogebra. The control group consisted of thirty students while the experimental group was composed of twenty-nine students. All the students were in 12th grade during the 2020-2021 school year.

5.2.2 Research Goal

The objectives of this study are as follows:

1. To assess mathematical modeling skills among high school students.
2. To investigate the effects of dynamic mathematics software Geogebra on students' skills in mathematical modeling.
3. To identify how students interact in a Geogebra environment during modeling activities.

5.2.3 Sample and Data Collection

In 2020-2021, this study was undertaken in a private high school in the city of Kenitra (Western Morocco). Two math science classes (12th Grade) took part in the study. The experimental group profited from Geogebra software, while the control group received a classical constructivist method. The authors devised Geogebra's activities for this purpose (see the example of cooling temperature in this paper). A Wi-Fi connection and laptop PCs were provided for the experimental group. The two classes were similar in terms of their academic levels in general and in mathematics in particular (twenty-nine students in the experimental group and thirty students in the control group). Both groups took a pre-test before starting the study. The study lasted five weeks. At the end of the five weeks, both groups took a post-test.

5.2.4 Study variables

In this study, there are two types of variables: independent and dependent. With regard to independent one, it is the use of GeoGebra. As for the dependent variable, we have used two. The first dependent variable was the results of the students on the post-test which consisted of two different modeling problems. A scoring rubric was used to assess each step of the modeling process. The second dependent variable was students' attitudes toward mathematical modeling. The scale is divided into four domains (Enjoyment, self-confidence, value and motivation).

5.2.5 Measurements

The pre-test and the post-test

The pre-test was about math prerequisites for grade 12 and the post-test was in the form of two math modeling problems. The assessment used in the study comes from the pedagogic orientations of the Moroccan mathematics program (MEN, 2007). The teaching activities were developed by the authors of this study. Both groups followed the same activities, the only difference being the use of GeoGebra by the experimental group.

A Scoring grid Using Common Core State Standards for Mathematics (CCSSM) for Modeling Cycle (Leong, 2012)

To assess the modeling tasks, we adopted a scoring grid adapted from Leong (2012) (Table 3. Modeling Cycle Scoring Rubric). The grid was developed after considering the important checklist that is required in each process of the CCSSM modeling cycle. Each procedure used a 5-point rating score. Possible rating scores ranged from 0 to 4 and were assigned as follows: 0. Incomplete, 1. Below acceptable, 2. acceptable, 3. Good, 4. Excellent.

Each process was based on the importance of the process being evaluated during the modeling cycle. The extremely important steps in the modeling process were formulating a model and interpreting the results. For example, in the "Identifying Variables section, the maximum possible score was 12 points since each of the three items gets a 0 to 4 score with a weight of 1.

Assessing the Affective Domain of Mathematical Modeling (Leong, 2012)

Attitude Scale can be created to measure attitudes towards mathematical modelling (Table 4. Affective Test Domains). Each domain contains several items and students can rate the items with a

5-point Likert scale from Strongly Agree to Strongly Disagree.

5.2.6 Intervention

The students in the experimental group used GeoGebra during an exploration session (1 hour) to familiarize themselves with the tool and learn about the different functionalities. On the first day of the study, both groups took the pre-test. Then, four sessions took place, two per week. Each session lasted two hours with a total of eight hours of lessons and exercises related to differential equations and their uses in modeling activities. On the last day of the study, both groups took a post-test. The data was based on the pre-test and post-test results of the experimental group and the control group. The researchers utilized two grids to measure students' modeling skills and attitudes towards modeling activities with differential equations.

5.2.7 Data analysis

Independent samples t-tests (control and experimental group) were performed to determine significant differences in pre-test and post-test scores. We also examined the normality and homogeneity of the score variances. A Chi-square test was used to verify the similarity of the two groups (control and experimental) in the pretest, at the start of the study.

6. Results

6.1 Student performance in the Pretest

To study the effects of integrating GeoGebra into the mathematical modeling course, we formed two groups of students. The control group followed the mathematical modeling course using a traditional approach (paper and pencil), the experimental group followed the same course with the contribution of Geogebra. To verify the equivalence of the two groups, we constructed a grade 12 pre-test. The test measured students' prerequisites in mathematics. Thirty items have been constructed to cover all the mathematical skills necessary to follow the course of mathematical modeling with differential equations. To verify the similarity between the levels of the students in both groups, a Chi-square test was carried out. Levels are defined in Table 2.

Table 2: Students' Performance in Pretest

Group Niveau	**Experimental Group**	**Control Group**	**χ2**	**p**
Advanced (24 - 30)	3	5	1.067	0.785
Proficient (18 - 23)	9	7		
Approaching Proficiency (12 - 17)	11	10		
Developing (6 - 11)	6	8		
Beginnin (0 - 5)	0	0		
Total	29	30		

The χ2 test indicates that there is no significant difference in the distribution of students in the two groups according to the different levels in mathematics ($\chi2 = 1.067$; $p = 0.785$).

6.2 Assessing modeling tasks

To measure the modeling skills of students in the two groups, experimental and control, we adopted an assessment grid. This grid evaluates the students' modeling process. Six steps in the modeling process have been defined: identification of variables in the model, formulation of the model, mathematical operations, interpretation of the results, validation or revision of the model and summary of the results (Table 3).

Table 3: Modeling Cycle Scoring Rubric

Process	Experimental Group (N = 29)		Control Group (N = 30)		t	p
	Mean	S.D	Mean	S.D		
Identifying Variables (12 Pts)	9.1	2.7	10.2	2.3	1.68	0.09
Formulating a Model (36 Pts)	28.2	4.5	25.8	5.1	1.11	0.26
Mathematical Operations (24 Pts)	21.3	3.6	16.7	6.1	3.51	< 0.01
Interpreting the Results (36 Pts)	31.7	4.1	25.1	7.3	4.26	< 0.01
Validating the Conclusion (24 Pts)	22.5	3.9	18.6	4.9	3.37	< 0.01
Reporting on Conclusions (8 Pts)	6.5	1.9	4.7	2.1	3.44	< 0.01

The first step in the modeling process includes the "Identifying Variables", the student must state the variables of the model, state the problem and the important characteristics. In this step, there was no significant difference between the two groups ($t = 1.68$; $p = 0.09$). Then in the "Formulating a Model" step, the student must create the model, state the hypotheses and describe the relationships between the variables. There was no significant difference between the two groups ($t = 1.11$; $p = 0.26$). In the "Mathematical Operations" section, it is about using mathematical operations correctly and analyzing the relationships between variables. In this step, there was a significant difference between the two groups ($t = 3.51$; $p < 0.01$). Geogebra has a set of tools that facilitate mathematical operations.

At the "Interpreting the Results" step, the aim was to evaluate the model and the solution obtained. The difference was significant between the two groups ($t = 4.26$; $p < 0.01$). The contribution of GeoGebra was considerable. The dynamic nature of GeoGebra allowed students to change the parameters of the model and observe the effects. The penultimate step, "Validating the Conclusion", the student had to revise the model according to the problem, interpret the solution according to the revised model, and improve the model. There was also a significant difference between the two groups ($t = 3.37$; $p < 0.01$).

In the final step "Reporting on Conclusions", the student summarized the results and answered the hypotheses. Also, in this step there was a significant difference in favor of the GeoGebra Group ($t = 3.44$; $p < 0.01$).

Overall, during the post-test, the control group that was taught using the traditional method scored lower than the experimental group that was taught using GeoGebra. This indicates the positive effect of the introduction of Geogebra on the learning of mathematical modelling.

6.3 Assessing students' attitudes towards mathematical modeling activities

The second grid assessed the affective domain of mathematical modelling. Using the 5-point Likert scale, and after the students completed the modeling tasks, their appreciation of the mathematical modeling activities was tested. The grid was divided into four areas: Enjoyment, Self-Confidence, Motivation and Value. The Likert scale was 5 points ranging from strongly agree to strongly disagree. The Enjoyment domain measures the student's desire and enjoyment in solving modeling tasks. The self-confidence domain measures how calm or stressed the student is during modeling activities. The value domain measures the interest and importance of mathematical modeling for the student. The fourth domain measures student motivation for modeling problems.

Table 4: Affective Test Domains

Domain	Experimental Group		Control Group		t	p
	Mean	S.D	Mean	S.D		
Enjoyment (12 Pts)	10.1	3.3	6.9	2.1	4.45	< 0.01
Self-Confidence (12 Pts)	9.7	2.5	7.7	3.0	2.77	< 0.01
Value (16 Pts)	14.2	3.1	10.3	2.7	5.15	< 0.01
Motivation (12 Pts)	11.3	2.9	9.4	2.5	2.69	< 0.01

The integration of Geogebra in modeling activities has a positive effect on students' attitudes towards mathematical modelling (Table 4).

In the "Enjoyment" dimension, the student was questioned about his/her enjoyment of performing modeling tasks and his/her curiosity to see the functioning of mathematics through modeling tasks. The difference was significant between the two groups ($t = 4.45$; $p < 0.01$). Another dimension in the attitude scale is "self-confidence". At this level the difference was highly significant between the two groups ($t = 2.77$; $p < 0.01$). In the GeoGebra environment, the student began modeling activities more easily. Regarding the value of mathematical modelling; i.e, the usefulness of the subject and of mathematics in general, the students of the experimental group appreciated the usefulness of mathematics more than the control group ($t = 5.15$; $p < 0.01$). Finally, the motivation of students in the experimental group was greater in the course of mathematical modeling activities ($t = 2.69$; $p < 0.01$).

7. Discussion

Throughout our present study, the opinions of teachers varied on the use of technology in the teaching of mathematical modelling. There are those who said students should learn the “basics” first. They suspect that the CAS (computer algebra system) can support a Blackbox approach. Other teachers believe that CAS can give students the opportunity to explore authentic and complex problems in mathematics. This technology is involved in almost all stages of mathematical modelling (Galbraith et al., 2003; Confrey et Maloney, 2007).

By using technology in mathematics education, it is possible to move from mathematical operations to planning and mathematical thinking. For example, dynamic geometry software (DGS) allows mathematical reflections because the solution is created by the software. These technological tools allow experimentation, verification of solutions, and communication in the teaching of mathematics. In addition, abstract mathematical objects can be presented more easily. According to the Common Core State Standards for Mathematics (CCSSM), modeling connects classroom mathematics and statistics to everyday life, work, and decision-making. When creating mathematical models, technology is valuable for varying assumptions, exploring consequences and comparing predictions with data (NGA, 2009).

This study aimed at identifying the effect of using Geogebra in learning mathematical modeling on the achievement of the twelfth-grade students and their attitudes towards its use. We used T-Tests to answer our research questions. The first question was whether the use of GeoGebra develops modeling skills in students. We have defined these skills according to the steps of the modeling process. For the first two steps, Identifying Variables and Formulating a Model, there was no difference between the control group and experimental group. For the rest of the steps, Mathematical Operations, Interpreting the Results, Validating the Conclusion, and Reporting on Conclusions, GeoGebra's contribution was important.

Based on the independent t-test results, there was a significant difference between the two groups. The group with GeoGebra performed better in the post-test compared to the group that received traditional education. Kutluca (2013) studied the effect of GeoGebra software on students' level of understanding of Van Hiele geometry. Students had the opportunity to test and build their own knowledge. They had the opportunity to actively participate in the teaching process (Kutluca, 2013). On the other hand, the study by Ljajko, Mihajlović and Pavličić (2010) found that students who used GeoGebra showed better results in problem solving, but lacked a deeper understanding of mathematical concepts (Ljajko et al). Saha et al. (2010) studied the effects of GeoGebra on geometry learning. They found that GeoGebra presented different levels of visualization to students to help them develop their understanding. Students use GeoGebra during mathematical modeling activities in different ways, drawing, calculating, measuring, constructing and experimenting (Siller, H.S., & Greefrath, G., 2010). Constructing and drawing when moving from the real model to the mathematical model. Calculating and experimenting when moving from the mathematical model to the mathematical results (Greefrath, G., & Siller, H.S., 2018). By using GeoGebra in modeling with differential equations, the student can overcome his/her learning difficulties. Through interactive animated applications, the student can learn different representations, symbolic, numerical, and graphic of mathematical concepts and move forward and backward between these modes (Funes, J. O., & Valero, E., 2018; Caligaris et al, 2015).

The second research question concerned the attitude of students towards mathematical modeling activities. We used a scale of students' attitude towards modeling activities. We compared the two groups of our research (Experimental and Control). We asked the question: Are students in the experimental group more motivated by modeling problems compared to the control group? A comparison of the means showed that the modeling activities with GeoGebra had positive effects on the students' attitude towards modeling activities. The introduction of technology in the mathematics course is a practical tool to facilitate student learning (Boggan et al., 2010; Latifi et al., 2021). In addition to its positive impact on student achievement, GeoGebra software also has a positive effect on student perception of mathematics. It increases their enthusiasm, confidence and motivation. It also allows students to think critically and creatively (Arbain, N., & Shukor, N. A., 2014; Yılmaz et al, 2010). The digital environment encourages students and teachers to engage in learning and teaching (Ozdamli, Mus and Nizamoglu, 2013; Korenova, 2012).

8. Conclusion

Based on the analysis of normative documents, scientific, pedagogical and, methodological literature, the authors have developed and introduced a method of teaching of mathematical modeling elements with Geogebra. With this software, students become more engaged in their mathematics learning. In this study, the modeling processes for real-life problems were described and formulated. Mathematical modeling in high school (grades 10–12) with Geogebra can develop the relatively modest modeling typically done in class to embrace more real-life situations and resolve more interesting and complicated problems. Newton's cooling law is easily understood, but using different methods to create solutions can help students understand that mathematics is more than just a single solution. The article can be useful for teachers of mathematics and will improve the overall students' perceptions of mathematics.

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