



Research Article

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Models Connecting Points on Pupils' Achievement in Rational Numbers

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Abstract

The study investigated pupils' achievement in rational numbers, using constructivist models and traditional approach of instruction as connecting points between pupils' prior knowledge of whole numbers concept and the new concept. Pre-test post-test non-equivalent control group quasi-experimental research design was adopted. A research question guided the study and was answered using descriptive statistics; and a formulated corresponding null hypothesis was tested at 0.05 level of significance, using analysis of covariance (ANCOVA). A sample of 103 pupils was used for the study. A test instrument titled Rational Numbers Achievement Test (RNAT) was developed, validated and used to generate data. The instrument had 0.74 reliability index of internal consistency through the use of Kuder Richardson formula 21. The results showed that different types of representation are central to conceptual understanding, and are able to resolve pupils' difficulties and misconceptions about rational numbers. Based on these findings, it was recommended that constructivist models of instruction should be part of the main instructional approaches for the teaching-learning of mathematics at the Basic Levels of the Nigerian school system.

Keywords: *Rational numbers; conceptual understanding; pupils' achievement; numbers place-values; active participation; learning materials*

1. Introduction

Observations have shown that many pupils get confused between rationals and whole numbers notations. Pupils' lack of proper understanding of these two number systems has led to some misconceptions, difficulties and poor achievement. In the concept of whole numbers, achievement depends on pupils acknowledging that, it is the digits with the highest value that contributes the most to the magnitude of a number. Digits that make up whole numbers are grouped in threes from right to left alongside the group name such as: units, hundreds, thousands, millions, to mention just a few. These groupings form the positions or place-values of such digits. Achievement of pupils in rational numbers is hindered when whole numbers place-values are not extended to include their equivalents in rational numbers.

Numbers expressed as fractions are called rational numbers; which are thought of as representing one or more of equal parts of a unit. To divide any integer (that is, a positive or negative

number) by any other integer except zero brings about the idea of rational numbers. These are numbers of the form x/y ; where x and y are integers and y is not equal to zero ($y \neq 0$). Also, any integer written with unit denominator is a rational number. For example, $5/1$, $10/1$, $25/1$ are all rational numbers. Decimal fractions, ratios, rates or percent are special kinds of rational numbers (or fractions) with denominators of 10, 100, 1000, to mention just a few. Examples include:

1. $5/10$ read as five-tenths, can be written in decimal form as 0.5;
2. $10/100$ read as ten-hundredths, can be written in decimal form as 0.10. This is also known as 10 percent (10%);
3. $25/1000$ read as twenty-five-thousandths, can be written in decimal form as 0.025; to state just a few.

Achievement of pupils based on rational numbers, is the measure of knowledge or skills possessed by the individual pupil, depending upon the circumstances of the teaching-learning process and the environment in which the pupil operates. Nwana (2007) opined that conditions of learning differ from pupil to pupil in that, when the conditions of learning are uniform, the more capable pupil will achieve more than the less capable, while those of equal ability will achieve the same. The concept, achievement is a measure or score of an individual's degree of accomplishment or learning in a particular skill, task or subject (Nwana, 2007; Ukwuije & Opara 2012). Knowles as cited in Joe, Kpolovie, Osonwa and Iderima (2014:203), defines achievement as "knowledge attained or skills developed in school subjects usually designed by test scores or marks assigned by the teacher or both". Testing of pupils' achievement on rational numbers therefore, is to determine levels of achievement so far attained by them.

Pupils' achievement in rational numbers have numerous benefits to various disciplines, particularly in mathematics and its related disciplines, in business, accounting and in other life activities. Some of these benefits include serving the purpose of describing parts of a whole in monetary applications; document literacy such as in reading and understanding frequency distributions, graphs, charts and interpretation of data; computing time and timing of athletes; measuring of computer speed; presentation of pupils' and students' performances on examinations; calculating bank interest rates; to mention just a few. Despite these benefits derived from rational numbers, researches (Bennett, 2009; Flockton, Crooks, Smith & Smith, 2006; Irwin & Britt, 2004; Moody, 2007, 2008; Saxe, Edd, Taylor & Gearhart, 2005; Stafylidou & Vosniadou, 2004; Steinle & Stacey, 2001; Steinle, 2006, 2004; Tzur, 2007; Ward & Thomas, 2007) have shown that, there are misconceptions pupils and students experience when learning rational numbers which had contributed to their poor achievement in this mathematical concept. Moody (2008) observed that when pupils or students or even adults write rational statements such as, $1/2 + 1/3 = 2/5$; $1.25 > 1.5$; or $1.4 + 1.25 = 2.29$ (instead of $1/2 + 1/3 = 5/6$; $1.25 < 1.5$ and $1.4 + 1.25 = 2.65$), they are simply drawing from existing whole number procedures that have proven effective in the past. According to the researcher this is as a result of pupils' difficulties in understanding place-values when presenting rational numbers and lack of understanding of what symbols such as $2/3$ (two-third) or the decimal point(.) stands for in rational quantities.

Despite the wide-spread knowledge about the deep-seated difficulties pupils and students encounter when studying rational numbers, there are evidences to suggest that little change had taken place in terms of their achievement as they use whole numbers place-values as referent system, drawing upon them as prior understanding. The points of connection between pupils' prior understanding of whole numbers concept and that of the rational numbers will depend on the instructional models involved in linking their previously held ideas with that of the new concept. This will involve activities within a learning zone that can introduce pupils to working with the new concept (rational numbers), by reversing their misconceptions and false analogies previously drawn from the concept of whole numbers. Such instructional activities will help to expose the underlying place-values structure of the rational number system, and for making explicit connection between its many different facets.

These points of connections are the teaching methods – the traditional approach of instruction

and the constructivist models used as the independent variables for the study. Although, these teaching methods of learning mathematics ensure the breaking down of activities of particular learning concept(s), the question that has not been satisfactorily answered is: What help to make positive changes about the deep-seated difficulties pupils encounter when studying rational numbers? The purpose of this experimental study therefore, is to proffer answer to this unresolved pupils' difficulty. Hence, the study provided answer to the following research question.

2. Research Question

What is the difference between the mean achievement scores of pupils taught rational numbers using constructivist models, traditional approach of instruction and the control group?

3. Hypothesis

A Corresponding null hypothesis was postulated and tested at 0.05 level of significance for tenability; thus:

There is no significant difference in the achievement of pupils taught rational numbers using constructivist models, traditional approach of instruction and the control group.

4. Literature Review

The study by Steinle and Stacey (2001) reported the difficulties some statistics students had when statistical software calculates a p-value of 0.0493 for a test. The researchers found out that a student may know that p equals 0.0493 (that is, $p = 0.0493$), but often does not know if such value is less than or greater than the cut-off point of p equals 0.05 ($p = 0.05$). Also, Steinle (2004), quoting Guardian Weekly of June 28, 1997 during the opening of the G. 8 Summit, reported how former President of the United States of America (USA), Bill Clinton confused the population of USA, which is slightly less than five percent (5%) of the world's population, for less than one-fifth ($1/5$). This misconceptions and difficulties of rational quantities, according to Stafylidou and Vosniadou (2004) are due to, first, the principle of relative size (that is, density), that stands in contrast to the presupposition of discreteness. In other words, rational values lack unique symbols and transparency, which is prevalent in whole number system. Secondly, operations involving rational numbers contravene the patterns that hold true for whole number system, where each symbol (number) precedes and follow a unique successor; one before and one after.

As instructors introduce this concept, Moss and Case (1999), observed that rational numbers are not differentiated from whole numbers (which can cause misconceptions and difficulties). Secondly, Rational numbers notations, especially the decimal point notation, entail specific difficulty by its own right. When ignored, makes it more problematic for pupils and students to make sense of the conceptual understanding of decimal fractions. (Hiebert, as cited in Moss & Case, 1999). Ni and Zhou (2010) also reported how pupils use the single-unit counting scheme applied to whole numbers to interpret rational numbers. Hence, there are bound to be misinterpretations of rational numbers that are ordered and continuous, for whole numbers that represent discrete numbers.

Models of instruction such as the traditional or conventional approach, that can be used to resolve pupils' difficulties in the learning of rational numbers, ensure the breaking into sequence of activities of particular learning concept(s). With regards to mathematics instruction, the traditional approach seeks the clarification of the nature of mathematics constructs, like the rational numbers concept. But, acquiring knowledge in mathematics in this approach to learning, is only based on computational competency (Moody, 2008), rather than the competency that transcends initial classroom settings. The researcher asserts that, pupils have difficulties in connecting proficiency previously acquired through the conventional approach of instruction to new situations, where necessary. Adepoju (2004) observed that mathematics at the Basic Level of the Educational System in

Nigeria is still dominated to a large extent by unmotivated drills and purposeless skills, characterized largely by rote learning (the process of repeating a piece of information until you remember it, rather than by understanding its meaning). Exline (2004) also observed that in traditional approach to learning, pupils are faced with a vast amount of information to memorise; much of which seems irrelevant outside the classroom. Morton (2004) also reported that, pupils' learning of mathematical concepts using traditional approach of instruction focuses on mastery of computations, with less emphasis on conceptual understanding, development of skills and the nurturing of inquiring attitudes. It could be argued that, this approach is more concerned with the preparation of pupils for the next level and in-school success, rather than helping pupils learn how to learn throughout life.

Achufusi and Mgbemena (2012) observed that, lecture method (one of the traditional approach of instruction) discourages initiative, curiosity and creativity in students and does not offer them opportunity to effectively interact with peers and learning materials. Both researchers argued that, this method has resulted in student's loss of interest, reduced participation in class and poor achievement in their academic work. Also, Obanya (as cited in Ogbodo, 2011) observed that, students retain only 5% of what is taught through lecture method. According to Schwerdt and Wuppermann (2011), students' attention quickly wares out while information received tend to be forgotten easily when students are passive during lectures. Ogbodo (2011) therefore called for a need to introduce best practices and reform in pedagogy. The researcher opined that reliance on the conventional approach of instruction is now obsolete.

From personal experience, traditional approach of instruction reduces the inherent difficulties of mathematics, by providing information concerning the best learning sequences and the inter linkage. Although, in this method of instruction, pupils play passive and receptive roles, while the teacher's responsibility is to teach. In most cases, meaning and interpretations of mathematical terms, symbols and structures may be quite different between that of the teacher and the pupils. In this regard, effective communication will be defective if the mechanism of intended dialogue is not common to both parties. Although, conventional approach of instruction looks at the nature of mathematics concepts to reduce the inherent difficulties there in, advocates of alternative methods of instruction argued that, this approach over-emphasises memorisation and repetition, thereby failing to present mathematical concepts as creative or exploratory (Adepoju, 2004; Madu & Ogbonna, 2007; Major, 2001). Brooks and Brooks (as cited in Madu & Ogbonna (2007) observed that, pupils struggle to make connections in traditional approach of instruction where they see disparity, and accept as reality what their perceptions questioned.

Other approaches to learning, like the constructivist models of instruction, focus mostly on previously held ideas pupils bring into learning new knowledge or concept. Constructivist models of instruction is inquiry based and self-directed. This instructional approach emphasis that pupils should actively participate in the learning process, to enable them constructs their own understanding (Atherton, 2010; Hunter & Anthony, 2003; Madu & Ogbonna, 2007; Mathews, 2000; Moody, 2008; Sophian, 2008). According to Mathews (2000), constructivist models of instruction alert the teacher to the function of prior learning and extant concepts in the process of learning new materials, by stressing the importance of conceptual understanding as a goal of instruction, which fosters pupils' engagement in lessons. According to Moody (2008), Hunter and Anthony (2003), knowledge gained in constructivist models to learning refers to the individual experience through problem solving, constituted by conceptual structures, which in turn, constitute knowledge when pupils regard them as viable, in relation to their experiences. Constructivist learning requires purposeful discussions between students; both in small groups and in whole classroom discussions.

Comparing constructivist and traditional instructions in mathematics, Alsup (2004) found participants used for the experimental group based on constructivist methods to be more confident and actively involved in both small and whole classroom discussions. The students were found to enjoy working with one another. Also, the experimental group significantly out perform their counterparts in the control group. The experimental group participants analysed data collectively and experienced a significant decrease in mathematics anxiety, together with a significant increase in

mathematics teaching efficacy and autonomy. Kazemi and Ghoraiishi (2012) also found out that Problem-Based Learning (PBL) – a constructivist kind of pedagogy, was more effective than the traditional approach to students learning of mathematics (having mean difference significant at the chosen alpha of 0.05; $t(81) = 2.33, p < .03$).

In their studies, Moody (2008), Madu and Ogbonna (2007) reported that, constructivist models to learning mathematics promote students' achievement through participation in activities, learning by doing, and learning by understanding rather than rote/blind memorisation. These researchers assert that, this approach to learning is student- centred rather than teacher or subject-centred. Learning materials are provided by the teacher whose position is that of a facilitator. According to these researchers, students and pupils are provided the desired quality of teaching-learning situations and they utilised their cognitive abilities. Pupils and students are found to be eager in finding out something (a measure) when involved in activities, thereby encountering new learning skills.

Using constructivist models to learning, Brousseau, Brousseau and Warfield (2007), Irwin and Britt (2004) found that students were able to handle basic operations in rational numbers through games and appropriate principles such as, distribution or compensation to identify the part-whole nature of rationals. Also, the students were able to apply number of units, tenths, hundredths and thousandths. In another finding, Hunter and Anthony (2003) averred that, constructivist models to learning rational numbers promote the use of percentages as referent system to decimal fractions. According to these researchers, conceptual understanding of rationals can be stressed when various types of models are presented to pupils during instruction. Steinle and Stacey (2001) assert that, constructivist models to learning mathematics are able to resolve pupils' misconceptions about rational numbers.

5. Methodology

The study employed pre-test, post-test non-equivalent control group quasi-experimental research design, involving non-randomization of participants. Quasi-experimental research design according to Kpolovie (2010); Leedy and Ormrod (2010); Levy and Ellis (2011); Nworgu (2006); McMillan and Wergin (2002), is a type of experimental design in which the researcher has limited leverage and control over the selection of the study participants. According to McMillan and Wergin (2002), quasi-experimental research is that which manipulates "treatments" but does not use randomly assigned treatment groups. Levy and Ellis (2011), and Kpolovie (2010) also assert that, in quasi- experimental research, the researcher does not have the ability to ensure that the sample selected is as homogeneous as desirable. With this design, however, the researcher relies more on techniques instead of randomisation to control extraneous variables for the minimisation of the influences of factors that threatens internal validity.

A sample of 103 pupils was used for the study. Three intact Lower Basic four (4) classes were randomly selected through simple random sampling technique from 48 public schools in Ethiopia East Local Government Area of Delta State, Nigeria. Two classes consisting 34 pupils each were used for the experimental groups (the constructivist models and the traditional approach of instruction) while a class consisting of 35 pupils was used for the control group. The treatment groups consisted of 17 boys and 17 girls each while the control group consisted of 17 boys and 18 girls (making a total of 103 participants).

A test instrument titled Rational Numbers Achievement test' (RNAT) was used for both treatments and control group pre-test and post-test. Pupils' achievement was measured with the RNAT instrument. The instrument was constructed and validated by the researcher with the assistance of two other experts in the area of measurement and evaluation for the editing of the test items. The experts' corrections and recommendations were included in both test items used for pre-testing/item analysis and the final draft that was used for the study. The instrument consists of 30 items (multiple choice); ranging from comparison/changing of decimal fractions equivalents to decimals and vice versa, and carrying out of basic operations. The Lower Basic 4 scheme of work contents were used for the construction of the items. Initially, sixty (60) items were developed, and

administered to 100 pupils outside the sample for the study. Item statistical procedure (item analysis) was carried out to elicit the desired responses to each of the items. Items with difficulty index of 0.5 and above, and discrimination index of 0.4 and above were selected for the test final form. While those with less difficulty and discrimination indices were rejected or discarded. The RNAT instrument consists of two sections (A and B). Section A consists of pupils' personal data (name of school and class) while section B consists of the items with the instructions on how to answer the items.

To ensure a high content validity of the instrument, a table of specifications was drawn. The table of specifications shows the content areas of rational numbers (common fractions, decimal fractions, ratios and percentages) in the Lower Basic 4 scheme of work covered, cognitive levels and the number of items allotted to each content area. Percentage weights were given to each content area based on the periods allotted, and the six levels of cognitive domain.

After the final draft of the RNAT instrument had been compiled, it was administered to another group of 30 pupils that were also outside the sample used for the study. The data generated was analysed using Kuder Richardson formula 21 to determine the coefficient of internal consistency of the test items. The Kuder Richardson formula 21 is based on the mean and variance (σ^2) of the total score from all test items. A reliability index (r) of 0.74 was obtained.

Prior to the start of experimentation, the RNAT instrument was administered to both the treatment groups and control group participants as the pre-test for the study. The treatment was a short-term teaching experiment that lasted for four (4) weeks in the sampled schools. The treatment groups were exposed to constructivist models (using models such as measuring pipes, cut strings, water-cans, number lines, etc) and the traditional approach of instruction. The constructivist models (Experiment Group 1) involves presentation of learning materials and tasks; purely interactive, questioning, activities and recording of observations by pupils. While experiment group 2 (the traditional approach) had the normal conventional whole class method of instruction. No treatment for the control group. At the end of the period of experimentation, the RNAT instrument was re-administered to all 3 groups as post-test for the study.

6. Results

Research question: What is the difference between the mean achievement scores of pupils taught rational numbers using constructivist models, traditional approach and the control group?

The results are shown on table 1 below:

Table 1: Descriptive Statistics (Mean and Standard Deviation) of Pupils Taught Rational Numbers with Constructivist Models, Traditional Approach and the Control Group

Variable	n	Mean	SD
Constructivist Models	34	25.0294	2.5404
Traditional Approach	34	12.4706	1.5616
Control Group	35	10.3429	1.5894
Total	103	15.8932	6.7837

Computer software used: IBM SPSS statistics version 20

The results in table 1 showed mean and standard deviation for experiment group 1 (constructivist models of instruction) as 25.0294 and 2.5404 respectively; experiment group 2 (traditional approach of instruction) as 12.4706 and 1.5616 respectively; and that of the control group as 10.3429 and 1.5894 respectively. This is an indication that, there is difference between the mean achievement scores among the three groups.

Null Hypothesis (Ho): There is no significant difference in achievement of pupils taught rational numbers using constructivist models, traditional approach of instruction and the control

group.

Results of the Ho is shown in Table 2 below:

Table 2: ANCOVA Results of Pupils Taught Rational Numbers with Constructivist Models, Traditional Approach and the Control Group

Source of Variance	Type III Sum of Squares	df	Mean Squares	F	Sig
Between Groups:	4151.250	2	2075.625	1012.287	.000*
Within Groups	202.993	99	2.050		
Correction Total:	4693.825	102			

* P<0.05

Computer software used: IBM SPSS statistics version 20

Results in Table 2 shows Between Groups Type III Sum of Squares as 4151.250 and that of Mean Squares as 2075.625; $F(2,99) = 1012.287$, statistically significant at the chosen alpha level of 0.05. p-value of .000 taken as .001 is less than 0.05. Therefore, the Ho which states that, “There is no significant difference in achievement of pupils taught rational numbers using constructivist models, traditional approach of instruction and the control group” was rejected. This means that, there is significant mean difference in achievement of pupils among the three groups taught rational numbers (using constructivist models, traditional approach of instruction and the control group).

Specifications of the three groups differing from one another is presented in Table 3 below:

Table 3: Pairwise Comparisons of Pupils Taught Rational Numbers using constructivist Models, Traditional Approach and the Control Group

Teacher methods (i)	Mean difference (j)	Std. Error (I-J)	Sig	95% Confidence interval for difference	95% Confidence interval for difference	
					Lower bound	Upper bound
Exp. 1	Exp2	12.649*	.347	.000	11.959	13.338
	Contr.GP.	14.262*	.348	.000	13.572	14.952
Exp. 2	Exp 1	-12.649*	.347	.000	-13.338	-11.959
	Contr.GP.	1.614*	.349	.000	.921	2.306
Control GP	Exp. 1	-14.262*	.348	.000	-14.952	-13.572
	Exp. 2	-1.614*	.349	.000	-2.306	-.921

*The mean difference is significant at .05 level of significance

Computer soft ware: IBM SPSS statistics version 20

From Table 3, the mean difference between constructivist models (Exp. 1) and traditional approach of instruction (Exp. 2) is 12.649 with standard error of .347 and p-value = .000 taken as .001, having 95% confidence interval for difference between 11.959 and 13.338. Also, there is a mean difference of 14.262 between those taught, using the constructivist models and the control group; and a mean difference of 1.614 between those taught with the traditional approach of instruction and the control group. It is very clear from Table 3 that haven partialled out the effect of the pretest (the covariate), Exp. 1 (constructivist models of instruction) differs significantly at .000 (taken as .001) from Exp. 2 (traditional approach of instruction and the control group). Also, exp. 2 (the traditional approach of instruction) differs significantly at .000 (taken as .001) from Exp 1 (constructivist model) and the control group.

7. Discussion of Findings

The findings of the study showed that pupils taught rational numbers using constructivist models of instruction achieved higher mean scores compared with those taught with traditional approach. The

results also showed statistically significant difference between the two experimental groups (constructivist models and the traditional approach of instruction). It therefore implies that constructivist models of instruction are more effective in enhancing pupils' achievement in rational numbers.

This finding agrees with Alsup (2004); Brousseau, Brousseau and Warfield (2007); Hunter and Anthony (2003); Irwin and Britt (2004); Kazemi and Ghoraishi (2012); Madu and Ogbonna (2007); Steinle and Stacey (2002, 2001), who assert that, the use of constructivist models in learning mathematics is able to resolve pupils' and students' difficulties and misconceptions as students were able to handle learning materials confidently, particularly in rational numbers. Kazemi and Ghoraishi (2012); Madu and Ogbonna (2007) report that constructivist models promote students' achievement through participation in activities, learning by doing, working in small groups and learning by understanding rather than rote/blind memorisation (as in the case of the traditional approach of instruction). This finding equally agrees with Alsup (2004) and Moody (2008) who found that constructivist models of instruction develop and utilise pupils' cognitive abilities, decrease mathematics anxiety, promote team work and eagerness in finding out something (a measure) as activities are involved, thereby encountering new learning skills. Pupils were able to handle basic operations in rational numbers (Brousseau, Brousseau & Warfield, 2007; Irwin & Britt, 2004). This finding therefore confirms the report of Ogbodo (2011), who calls for a need to introduce best practices and reform in pedagogy other than the conventional method of instruction in the Nigeria school system.

8. Conclusion

The study investigated pupil's advancement in rational numbers using constructivist models and traditional approach of instruction as connecting points between pupils' prior knowledge of whole numbers concept and the new concept. Pre-test post-test non-equivalent control group quasi-experimental research design was employed.

The study was guided with research question and hypothesis. After data analysis, it was found that different types of representation were central to conceptual understanding and were able to resolve pupils' difficulties and misconceptions about rational numbers. These represent the connecting points between pupils' prior understanding of whole numbers concept and new concept. The study offers evidence that constructivist models of instruction can enhance achievement in mathematical concepts and therefore, their use is necessary in pupils' learning process that can build solid foundation in mathematical sciences needed for technological national development. The features of constructivist models suggest that they can be implemented in the Nigeria existing Basic School settings.

9. Recommendations

- 1) Curriculum planners, educational administrators and policy makers should fully recommend constructivist models as one of the teaching-learning approach at the Basic levels of the School System, particularly for the teaching-learning of mathematics.
- 2) Government should provide the models to be used for this pedagogy at the Basic school levels to encouraged mathematics teachers to put this teaching method to practice.
- 3) Teachers should be trained on the proper use of these models and pedagogy, since this pedagogy only involves presentation of materials and/or tasks, interaction, questioning, and recording of facts as against the conventional model of instruction.

References

- Achufusi, N. N., & Mgbemena, C. O. (2012). The effect of using mastery learning approach on academic achievement of senior secondary school II Physics students. *Elixir International Journal*. Retrieved 26th January, 2020 from <http://www.elixirpublishers.com>.
- Adepoju, J. A. (2004, July 15). How to teach mathematics effectively. *Vanguard*, 21 & 27.
- Alsop, J. (2004). A comparison of constructivist and traditional instruction in mathematics. *Educational Research Quarterly*, 28(4), 3-17.
- Atherton, J. S. (2010). Learning and teaching: constructivism in learning. Retrieved 5th December 2019 from <http://www.learningandteaching.info/learning/constructivism.htm>.
- Bennett, Jr. A. B. (2009). NCTM's standards on teaching decimals. [online]. Available: ABB@cisunix.unh.edu.
- Brousseau, G., Brousseau, N., & Warfield, U. (2007). Rationals and decimals as required in the school curriculum part 2: From rations to decimals. *Journal of Mathematical Behaviour*, 26(4), 281-300.
- Exline, J. (2004). *Workshop: inquiry-based learning concept classroom*. [online]. Available <http://www.thirteencelebration.org>.
- Flockton, L., Crooks, T., Smith, J., & Smith, L. (2006). *NEMP report 37: mathematics assessment results 2005*. New Zealand: Ministry of Education.
- Hunter, R., & Anthony, G. (2003). 'Percentages: a foundation for supporting students understanding of decimals.' In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.), *Proceedings of the 26th Conference of the Mathematics Education Research Group of Australasia*, 2, 452-459.
- Irwin, K., & Britt, M. (2004). 'Operating with decimal fractions as a part-whole concept.' In I. Putt, B. Faragher & M. Mclean (Eds.), *Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia*, 1, 3 12-319. Townsville: MERGA.
- Joe, A. I., Kpolovie, P. J., Osonwa, K. E., & Ideriman C. E. (2014). Modes of admission and academic performance in Nigerian universities. *Merit Research Journal of Education and Review*, 2(9), 203-230.
- Kazemi, F., & Ghoraiash, M. (2012). Comparison of problem - based learning approach and traditional teaching on attitude, misconceptions and mathematics performance of university students. *Procedia - Social and Behavioral Sciences*, 46, 3852-3856.
- Kpolovie, P. J. (2010). *Advanced research methods*. New Owerri: Springfield Publishers Ltd.
- Leedy, P. D., & Ormrod, J. E. (2010). *Practical research: planning and design* (9th ed.). Upper Saddle River, NJ: Prentice Hall.
- Levy, Y., & Ellis, T. J. (2011). A guide for novice researchers on experimental and quasi-experimental studies in information systems Researcher. *Interdisciplinary Journal of Information, Knowledge and Management*, 6, 151-161.
- Madu, B. C., & Ogbonna, C. C. (2007). Effect of a constructivist instructional approach on senior secondary students' achievement in mathematics in Nigeria. *DELSU Journal of Educational Research and Development DJERD*, 6 (1), 106-120
- Major, C. H. (2001). Assessing the effectiveness of problem-based learning in higher education: lessons from the literature. *Academic Exchange Quarterly*, 5(1).
- Mathews, M. R. (2000). 'Constructivist in science and mathematics education.' In D. C. Philips (Ed.), *National Society for the Study of Education, 99th Yearbook*, 161-192 Chicago. University Press.
- McMillan, J. H., & Wergin, J. F. (2002). *Understanding and Evaluation Educational Research, Second Edition*. New Jersey: Pearson Education, Inc.
- Moody, B. (2007). *Connecting the points*. Unpublished dissertation. University of Waikato, Hamilton, NZ.
- Moody, B. (2008). *Connecting the points: An investigation into students learning about decimal numbers*. [online]. Available: <http://waikato.researchgateway.ac.nz/>
- Morton, J. (2004). *Workshop: inquiry-based learning concept classroom* [Online]. Available: <http://www.thirteencelebration.org>.
- Moss, J., & Case, R. (1999). Developing children's understanding of rational numbers: a new model and an experimental curriculum. *Journal for Research in Mathematics Education*, 30(2), 120-147.
- Ni, Y., & Zhou, Y. (2010). Teaching and learning fraction and rational numbers: the origins and implications of whole numbers bias. *Educational Psychologist*, 40(1), 23-52.
- Nwana, O. C. (2007). *Introduction to educational research*. Ibadan: HEBN Publisher Plc.
- Nworgu, B. G. (2006). *Educational research: basic issues and methodology (second & enlarged edition)*. NSUKKA: University Trust Publishers.
- Ogbodo, C. M. (2011). Policy and reformation of education in Nigeria. *Research in Education: A Research Journal*, 17(1), 1-6.

- Saxe, G. B., Edd, V., Taylor, C. M., & Gearhart, M. (2005). Representing fractions with standard notation: a development analysis. *Journal for Research in Mathematics Education*, 36(2), 137-157.
- Schwerdt, G., & Wuppermann, A. C. (2011). Is traditional teaching really all that bad? a wetin – student between – subject approach. *Economics of Education Review*, 30, 365–379.
- Sophian, C. (2008). 'Rethinking the starting point for mathematics learning.' In O. Saracho & B. Spodek (Eds.), *Contemporary Perspectives on Mathematics in early Childhood Education*, 21-44. Charlotte, N. C: Information age Publishing.
- Stacey, K., Helme, S., & Steinle, V. (2001). Confusions between decimals, fractions and negative numbers: a consequence of the mirror as a conceptual metaphor in three different ways. [online]. Available:<http://extranet.edfac.unimelb.edu.au/DSME/decimals/SLMmversion/backinfo/refs/PME2001.pdf>.
- Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, 14(4), 503-518.
- Steinle, V., & Stacey, K. (2001). 'Visible and invisible zeros: sources of confusion in decimal notation.' In J. Bobis, R. Perry & M. Mitchelmore (Eds.), *Proceedings of the 24th Annual Conference of the Mathematics Research Group of Australasia*, 2, 434-441. Sydney: MERGA.
- Steinle, V., & Stacey, K. (2002). 'Further evidence of conceptual difficulties with decimal notation.' In B. Barton, K. Irwin, M. Pfannkuch & M. O. J. Thomas (Eds.), *Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia*, 2, 633-640. Auckland: MERGA.
- Steinle, V. (2004). Teaching and learning about decimal numbers. [online]. Available: <http://www.v.steinle@unimelb.edu.au>
- Steinle, V. (2006). Importance of decimal numbers. [online]. Available: <http://www.edfac.unimeib.edu.au/sme/>
- Tzur, R. (2007). Fine grain understanding of students' mathematical understanding: participatory and anticipatory stages in learning a new mathematical conception. *Educational Studies in Mathematics*, 66(3), 273-291.
- Ward, J., & Thomas, G. (2007). What do teachers know about fractions? In *Findings from the New Zealand Numeracy Development Projects 2006*, 128-139.
- Ukwuije, R. P. I., & Opara, I. (2012). *Test and measurement for teachers*. Alakahia-Akpor: Chadik Printing Press.