Research Article

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Analysis of the Institutional Relationship of the Modeling Activity in a Moroccan High School Textbook

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Abstract

To identify the institutional relationship to modeling as an element of algebraic thinking, this article aims at analyzing the algebraic potential residing in the modeling activities that appear in the school textbook "Al-Moufid"; referring to the Anthropological Theory of Didactics (TAD) developed by Chevalard(1999) and to the Epistemological Reference Model of Algebraic Thinking (MERPA) proposed by Najar et al.(2021). This official textbook is for 1st-year secondary Moroccan students (Age 12 to 13), as a level in which the arithmetic-algebra transition is manifested. In this institution, the pedagogical guidelines treat modeling as a skill to be developed in students without making an explicit definition or a link with the development of algebraic thinking. This analysis shows the considerable presence of modeling activities but with insufficient algebraic potential. On the other hand, several proposed modeling activities do not allow students to go completely through the modeling process as described by Chevallard (Chevallard, 1989).

Keywords: *modeling, algebraic thinking, algebraic potential, institutional relationship*

1. Introduction

Teaching algebra represents a real challenge for teachers as well as for didactics researchers all over the world. Most of the difficulties of algebraic order noted in students in the second year (Aged 12 to 13 years) are due to the transition of arithmetic-algebra between two fields which are separated in the traditional didactic approaches. Indeed, according to these approaches, algebra is considered a mathematical domain that generalizes arithmetic properties and operations. The mathematical organization adopted in Moroccan middle school does not specify a specific domain for algebra. It is limited to mentioning the literal calculation as a lesson that is part of the domain of numerical calculation "In numerical calculation. The goal is to master operations on relative decimal numbers, on rationals, and on squthis manual and which onef literal calculation (development and factoring techniques) and the resolution of equations and inequations" (men,2007) (pedagogical gidlines, 2009: 23). The textbook that represents the corpus of our analysis, "Al-Moufid, 2020," is considered a projection of the official pedagogical orientations. It is approved by the Ministry of National Education, Vocational Training, Higher Education, and Scientific Research. Modeling is mentioned only once in secondary school pedagogical guidelines as a capacity to be developed in students. Our objective is to analyze the knowledge taught in relation to algebraic modeling by focusing on the student's manual. The teacher's guide contains only directives for implementing lessons without any indication of didactic intention that justifies the authors' choices. In this article, we attempt to provide some answers to the following questions:

1. What knowledge to teach about modeling in the textbook (Al-Moufid, 2020) in terms of tasks?
2. What is the algebraic potential of the modeling activities proposed in the textbook (Al-Moufid, 2020)?
3. What is the influence of the modeling process on the algebraic potential of the chosen task?

The choice to analyze modeling is since it is considered one of the fundamental intellectual processes that allow the development of algebraic thinking, such as problem-solving (Abouhanifa, 2021; Moukhliss et al., 2022) and generalization (Ennassiri et al., 2022; Squalli, 2021).Our analysis refers essentially to the TAD proposed by Chevallard (1989) and to the praxeological reference model of algebraic thinking (MERPA) proposed by Najar et (2021)adopting the methodology related to analyzing official programs and textbooks used in the work of the OIPA(Squalli, 2020)

2. Theoretical Frameworks

2.1 Mathematical modeling

Chevallard defines mathematical modeling as "the mathematical study of extra-mathematical" or "intra-mathematical" systems(Chevallard, 1989). The modeling process goes through three steps, according to Chevallard:

1. We define the system we intend to study, specifying the "aspects" relevant to the study we want to make of this system, that is, the set of variables by which we cut it up in the domain of reality where it appears to us. We will designate these variables by the letters x, y, z, a, b, c, etc.
2. The model is then constructed by establishing a certain number of relations, IR, IR', IR", etc., between the variables considered in the first stage, the model of the system to be studied being the set of these relations.
3. The model thus obtained is "worked on" to produce knowledge about the system under study. The knowledge that takes the form of new relationships between the variables of the system"(Chevallard, 1989).

The first two steps are linked to reality, while the third is entirely mathematical. Chevallard define two registers when modeling an intramathematical situation: the "mathematized register," which is represented in the studied system, and the mathematical register, which represents the register in which "the modeling is conducted." The latter is the tool of modeling, while the former is the object of the modeling. We illustrate this "tool-object" dialectic, which corresponds to that of mathematics-mathematicized, with the following example:

Let us seek the dimensions of a rectangle of perimeter 24 and area 20. It is a situation that belongs to the field of geometry, specialy the concept of the area and perimeter of the quadrilaterals that represent the "mathematized". This intra-mathematical situation could be modeled by two equations with two unknowns. If x is the width of the rectangle and y is the length, the following system is obtained: x+y=12 and xy=20. It is the model that represents the mathematical and is used as a modeling tool.

In step 3 of this modeling, the system is worked on to provide new knowledge related to the notion of the 2nd-degree equation, leading to the equivalent equation $x^2 - 12x + 20 = 0$.

Realistic Mathematics Education (RME) currently speaks of the process of "mathematization" which involves activities in which one engages to achieve generality, certainty, accuracy, or simplification (Barnes, 2005; Blum & Leiß, 2007)

Through a process of progressive mathematization, learners can create mathematical ideas, knowledge, and procedures. This stream distinguishes between two types of mathematization:

Horizontal mathematization: serves to represent a real situation with the help of mathematical symbolization, i.e., the transition from the real world to its mathematical representation. Vertical mathematization represents the pure mathematical treatment of the situation data represented in each register. Barnes represents these two processes in the following diagram(Blum & Leiß, 2007):

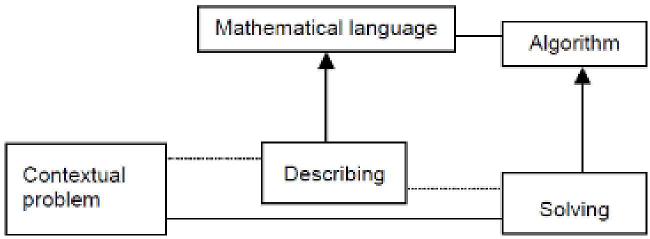


Figure 1: The representation of horizontal and vertical mathematization

According to Blum and Leiss (2007), the modeling process is based on a real-life situation model whose real model is already known or easily identifiable. It is a modeling cycle described by the following diagram:

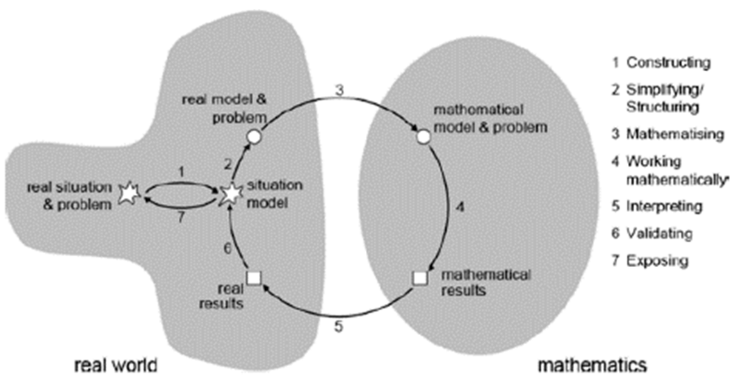


Figure 2: The representation of the modeling process according to the Blum and Leiss model

According to this perception, the modeling process goes through seven steps:

1. Moving from the real-life situation to the model situation, hypotheses are constructed about the problem to simplify the situation and extract the relevant data.
2. Moving from the model situation to the real-life model, we identify the variables that influence the real-life situation. Then, we name them and build relationships.
3. The transition from the physical model to the mathematical model: It is to mathematize the relevant variables and their relations, i.e., to represent these relations in mathematical language, then simplify them and make them as simple as possible, if necessary, by reducing their number and their complexity. Then, one chooses the appropriate mathematical domain to represent the situation. This stage allows the passage from the real world to the mathematical world.
4. Working on the mathematical model: We try to use heuristic strategies such as segmenting the problem, making relations with similar problems, reformulating the problem, and applying mathematical knowledge to solve it.
5. Give an interpretation of the mathematical results in the model situation.
6. Validate or invalidate the results in the model situation (if the results are not validated, return to step 2)
7. Communication of solutions in a real-life situation

2.2 *The anthropological theory of didactics (TAD):*

Yves Chevallard founded the anthropological theory of didactics, which places mathematics among "the whole of human activities and social institutions." As a result, the word "anthropological" is used to describe this theory, and each piece of mathematical knowledge is influenced by the institution where it was created and is still being changed.

2.2.1 *Basics of the anthropological theory of didactics:*

Yves Chevallard defines a set of concepts that govern his theory of which the notion of an object constitutes the starting point. Indeed, "any material or immaterial entity that exists for at least one individual"(Chevallard, s. d.-a)is an object, including people, numbers, symbols, excellence, dignity, equations...

Institution: it is the space that gathers objects of different types, individuals, objects, and personal relations that an individual makes with a given object of the institution. We illustrate the concept of an institution by the school, or as in our case study, the college cycle represents an institution.

In each institution I, every individual occupies a position; in our case, a position other than a teacher's is held by the pupil. To each X and an object of knowledge O, we associate a personal relationship $R_I\ (X, O)$ which gathers all the interactions of X towards the object O (to handle it, to retain it, to apply it ...). In the same way, for each institution and an object of knowledge O one defines an institutional relationship$R_I\ (O)$. A subject X is ideal for the institution I in relation to the object O if his personal relationship $R_I\ (X, O)$ is as close as possible to the institutional relationship to this object $R_I\ (O)$, « « une personne Y appelée à juger la connaissance qu'a une personne X d'un objet O ne sait guère qu'apprécier la conformité du rapport personnel $R_I\ (X, O)$ au rapport institutionnel $R_I\ (p, O)$), où p est la position que X est censée occuper au sein de I »(Chevallard, 2002). According to Chevallard, the person is the whole formed by the individual and the system of all his personal relations towards the objects of the institution. Consequently, the person changes with time but the individual remains invariant.

The object of knowledge in our case is modeling, and we are primarily interested in the institutional relationship to this object through a praxeological study of the "Al-Moufid" manual.

2.2.2 The praxeology model:

The praxeological model proposed by the TAD provides a set of mechanisms by which « l'assujettissement en position p d'une personne X à une institution I conduit à la formation, ou à la modification, ou à la confirmation du rapport personnel de X à un objet O, $R(X,O)$» .It is, in a way, an organization of the set of activities that the subject of an institution has to accomplish within this institution in order to improve his personal relation to a given object of knowledge in accordance with the institutional relationship related to the same object.

« Le rapport institutionnel à un objet, pour une position institutionnelle donnée, est façonné et refaçonné par l'ensemble des tâches que doivent accomplir, par des techniques déterminées, les personnes occupant cette position. C'est ainsi l'accomplissement des différentes tâches que la personne se voit conduite à réaliser tout au long de sa vie dans les différentes institutions dont elle est le sujet successivement ou simultanément qui conduira à faire émerger son rapport personnel à l'objet considéré. »(Bosch & Chevallard, 1999).

According to TAD, all human activity, including mathematical activity, consists in accomplishing a task t that is part of a specific type of task T and implemented by a technique τ (or several techniques). This technique is justified by a technology that also serves to question it and to produce other techniques related to the same task. At the end of this chain, we find the theory Θ that justifies the technology θ.

We then define the quadruplet $[T, \tau, \theta, \Theta]$, which represents a mathematical praxeology. In case of task that is related to mathematics, this quadruplet is made up of two blocks, the practical-technical block $[T, \tau]$, which refers to the practical part of praxeology, while the technological-theoretical block $[\theta, \Theta]$ refers to the theoretical part of praxeology. When the praxeological organization is centered on a single type of task it is said to be "punctual".

3. Research Methodology

The methodology that we propose consists of analyzing the official programs, in particular the Al-Moufid textbook, following the steps followed by the OIPA network and relying on the anthropological theory of didactics, which will be used to determine the praxeology relating to modeling in the targeted textbook.

Thus, our analysis methodology starts with the conception of a praxeological model in reference to algebraic thinking. Then, the identification of the official texts and the textbook were analyzed for our targeted institution, which is the first year of secondary school. Then, we analyze the mathematical praxeology related to mathematical modeling as a basic component of the development of algebraic thinking.

Choosing the textbook "Al-Moufid, 2020" is since it is part of textbooks that project the official pedagogical orientations approved by the Ministry of National Education, Vocational Training, Higher Education, and Scientific Research. It is among the textbooks that are frequently offered to students.

3.1 Praxeological Reference Model for Modeling (PRM)

Our Praxeological Reference Model for Modeling (MPR) is inspired by Najar et al's Praxeological Reference Model for Algebraic Thinking (MPRPA) (Najar et al.,2021) but focuses only on the regional praxeology of "modeling."

Thus, the proposed MPRPA views algebra as a collection of mathematical activities in which one or more operations (internal, external, binary, or n-ary laws of composition) intervene, which can be addition, subtraction, division... They are, however, only repeated a certain number of times. As a result, the activities considered are only those whose resolution necessitates performing at least one operation. These activities will then be classified based on two criteria: (1) their algebraic potential, and (2) the completeness of the modeling process.

The MPRPA proposed by Najar et al. (2021) is centered on three regional praxeologies, "Generalize," "Model," and "Compute," which serve as the foundation for the development of algebraic thinking because these components enable the mobilization of sophisticated algebraic reasoning based on generalization, analyticity, symbolization, reasoning about functional relations, and so on. etc.

1. The Regional praxeology « Generalization »:

The PMR "Generalization" collects all the tasks that lead to the student's generalization process and is divided into two local praxeologies: " Generalization of regularities " and "Generalization of rules, formulas, laws, and algorithms."

2. The Regional praxeology "Calculation" :

The PMR "Calculation" collects all tasks that require the student to perform one or more arithmetic operations ((+, − , × , ÷). This PMR has two local praxeologies: "Calculation on numerical expressions" and "Calculation on algebraic expressions."

3. The Regional Praxeology "Modeling":

The "Modeling" RMP brings together all the tasks that lead the student through the modeling process as described by Chevallard, according to which this process goes through the three stages described by Chevallard. This regional praxeology is broken down into three types of tasks:

M1: Modeling mathematical or extra-mathematical situations by numerical or algebraic expressions:

This type of task has been modified by adding algebraic expressions to the praxeology proposed by Najar et al.(2021)It gathers five types of tasks:

M1.1: Solve a situation that is modeled by a numerical or algebraic expression.

M1.2: Recognize a numerical or algebraic expression that models a given situation.

M1.3: Determine and represent a numerical or algebraic expression that models a given situation.

M1.4: Operate on a numerical or algebraic expression that models a situation to obtain results or conclusions related to the situation.

M1.5: Create a situation modelled by a numerical or algebraic expression.

M2: Modeling mathematical or extra-mathematical situations by equations:

This type of task also gathers five types of tasks:

M2.1: Solve a situation that is modeled by an equation.

M2.2: Recognize an equation that models a given situation.

M2.3: Determine and represent an equation that models a given situation.

M2.4: Operate on an equation that models a situation to draw results or conclusions related to the situation.

M2.5: Create a situation modeled by an equation.

M3: Modeling mathematical or extra-mathematical situations by functional relations:

M3.1: Solve a problem associated with a situation modeled by a functional relationship.

M3.2: Recognize a functional relationship modeling a given situation.

M3.3: Determine and represent in each register a functional relationship modeling a given situation.

M3.4: Operate on the functional model of a situation to draw results or conclusions related to the situation.

M3.5: Create a situation modeled by a given functional relationship.

Our analysis will be limited to the regional praxeology "modeling," even though a modeling problem can be the point of intersection for several regional praxeologies.

3.2 Characterization of the OMR Modeling

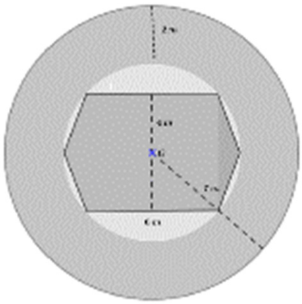
a. **Complete / incomplete modeling:**

Each modeling activity is complete when the problem modeled arises from a mathematical or extra-mathematical situation, and the instruction allows the student to go through the three stages of the modeling process described by Chevallard.

If, however, the model is given to the student, then the modelling is incomplete. We illustrate this characterisation with the following situation :

We propose the plan opposite, which represents a traffic circle in the city center. The hatched area is green space.

Approximate to the nearest unit the area of the green space portion?



This is an extra-mathematical situation in which the student's main task is to determine the model to be used. This model is not mentioned in the instruction for modeling the situation. He can use a mathematical model that calculates the area of a disc and the polygons that make up the figure, or he can approach the non-hatched parts and subtract them from the area of the large disc.

However, if the instruction is formulated as follows: calculate the area of the part devoted to the green space by decomposing the inner polygon into two isosceles trapezoids.
In this case, the model is given to the student, so the instruction does not allow the student to go through the modeling process, so it is an incomplete modeling.

b. High algebraic potential / Low algebraic potential:

For this criterion, we adopt the characterization defined in the MPRPA elaborated by Najar et al. (2021). According to this perspective, the algebraic potential represents the opportunities residing in the activity that is accessible by the student, and that leads him to use algebraic reasoning in which, the presence or absence of the letter is not determining.

"There are three levels of algebraic potential: zero, low, and high:

The first level is concerned with purely arithmetic tasks, that is, those that involve numbers that are all determined, and the task's realization is solely dependent on the naming quality of the numbers (their values) "(Najar et al., 2021). Connected problems are a good example of activities with zero algebraic potential, as in the following example:

Example1: Hajar has 200 DH. How much does she have left if she buys 2 comics at 45 DH each and a novel at 25 DH?

To solve this problemThe student can operates on the known quantities of the situation: 200 - (2 x 45 +25). It is envisaged that he adopts a purely arithmetic method by determining the operations to be carried out. We note that if a problem favors the understanding of an algebraic property based on the meaning of the expressions arising from the situation, it is regarded as having low algebraic potential. "The second level concerns tasks whose statements encourage the use of an arithmetic technique, or if the algebraic technique is beyond the student's reach." (Najar et al., 2021).

Example2: Hajar has twice as many pens as pencils. Knowing that there are 18 pencils and pens in Hajar's kit, does that mean Hajar has 6 pencils and 12 pens?

In this activity, the statement does not direct the student towards an arithmetic technique, but also does not favour an algebraic technique. Indeed, with the assistance of the verification, the student can use the arithmetic method: 12 is the double of 6, so the number of pens represents 2 times the number of pencils, and, we have 6 + 12 = 18, so Hajar has 6 pencils and 12 pens. The algebraic method considered is based on algebraic reasoning. If we note x the number of pencils, we will have the equation $3x = 18$. If the activity is proposed in the lessons that are programmed before entering formal algebra, this method will be out of reach for the student. Student can use analytical reasoning, operating on the unknown without representing it. In this method, the student operates 18 ÷ 3 to find the number of pencils; the number 3 is not in the data of the problem, but it represents how many times the number of pencils in Hajar's pencil case is repeated. If each pen is exchanged for 2 pencils, the unknown is silent and invisible. The third level represents tasks with strong algebraic potential. When "the statement of the task encourages the use of an algebraic technique, or if the algebraic technique is accessible to the student" (Najar et al., 2021).An example of tasks with strong algebraic is in the following problem: a 43-year-old man has three sons who are 5, 7, and 11 years old. After how many years will the father's age equal the sum of the ages of his three sons? Here, the task does not favor any trial/error arithmetic technique because it is a disconnected problem that requires the use of an algebraic technique.

4. Results

4.1 Habitats of 1st year middle school modeling elements

According to the didactic co-determination scale proposed by Chevallard (1989) at the discipline level and the four lower levels, we identify the habitats of the 1st year college level in the table below. Modeling is a transversal component and must occupy all domains. However, it is not explicit in any domain; only a few modeling elements (problem-solving; real-world problems...) are mentioned in the two domains. Which translates into an institutional void related to modeling at the level of early college.

Table 3: Habitats of 1st year middle school modeling elements

Domain	Sector	Theme	Subject
Numerical Activities	Equations	- Recognize the notion of the unknown. - Acquire some problem-solving techniques. - Check the validity of a solution. - Mathematization of problem situations.	- The equations aim to train students on mathematization and problem solving from their experiences through: - The identification and analysis of the problem data, the choice of the unknown and then the choice and implementation of the appropriate tools and techniques of resolution and finally the interpretation of the results found. To this end, students must be made aware of the importance of the unknown through diversified situations and then move on to the definition of the unknown, the equation, and the use of the properties of equalities in the solution of equations. - Diversified problem situations are needed so that the student touches the importance of using equations in solving problems and moves beyond the trial-and-error numerical methods used in elementary school to an algebraic phase.
Statistics	Statistics	- Read and interpret a statistical table, bar and pie chart and determine the statistical population. - Present statistical series in a table, chart, or graph. - Organize statistical data.	- The data must be real from the economic, social, or scientific sector and related to the students' daily life or to another school discipline. - Computer programs can be used according to the school's resources.

4.2 Characterization of the activities according to the algebraic potential

The results show that the textbook "Al-moufid" contains a total of 1117 tasks that involve operations. Among these tasks, there are 269 tasks that invite students to model a mathematical or extra-mathematical situation. The results are presented in the following table:

Table 4: distribution of task types according to their algebraic potential

	M1	M2	M3	Total
Zero potential (A)	60% (161)	1,5% (4)	6,3% (17)	67,66 % (182)
Low potential (B)	6,3% (17)	4,8% (13)	1,5% (4)	12,64 % (34)
High potential (C)	2,6% (7)	8,2% (22)	8,9% (24)	19,7 % (53)
Total	68,77% (185)	14,5% (39)	16,7% (45)	100 % (269)

The results presented in Table 4, related to the types of tasks, indicate that more than half of the activities fall under the regional praxeology "modeling". 67.66% (182) have no algebraic potential, 12.64% (34) have a weak potential, and only 19.7% (53) represent strong algebraic potential. This shows that most of the activities in the textbook where modeling is involved promote only arithmetic techniques.

However, the types of tasks most represented in this textbook in relation to the PML "modeling", are essentially of the type M1: modeling situations by numerical expressions. However, this type of task has the least number of tasks with algebraic potential in comparison with the other types M2 and M3, which reflects that a large part of the tasks of this type directs the students to apply arithmetic techniques based on calculation rules, this result also shows that the modeling activities that appear in this manual are almost exploited to the production of numerical expressions with the aim of performing calculations. In the second place, we find the type of tasks of the PML M3: modeling situations by functional relations 16,72% (45), of which more than half of these tasks (24) have a strong algebraic potential and almost only one-third (17) has no algebraic potential. In the last place, there is the type of tasks of the PML M2: modeling of situations by equations, which represents only 14.5% (39) of the totality of the tasks which fall under this regional praxeology. This type of task carries a significant algebraic potential compared to the two preceding types. Indeed, more than half (22) of these tasks have a strong potential, almost 90% (35) have at least a weak potential, and only 10% (4) are purely arithmetic. A comparison between the scores of the three PML of regional praxeology "modeling" leads us to deduce that the last two (M2 and M3) generally gather more tasks with algebraic potential than the purely arithmetical ones, which confirms that the mathematical or extra-mathematical situations which lead to an equation or a functional relation favor algebraic reasoning by the implicit or explicit presence of the notion of the variable or the unknown. However, the situations that lead to a numerical expression are almost empty in terms of algebraic potential. To identify precisely for each local praxeology, the types of tasks dominating in this manual, and which one have the strongest algebraic potential, we present in detail in Table 2, the distribution of the types of tasks according to their algebraic potential:

Table 5: Distribution of task types according to their algebraic potential

Potential Totaux Total	Modélisation												Total
	M1					M2			M3				
	M1.1	M1.2	M1.3	M1.4	M1.5	M2.1	M2.3	M2.4	M3.1	M3.2	M3.3	M3.4	
	13,01 % (35)	7,43 % (20)	44,98 % (121)	2,97 % (8)	0,37 % (1)	13,01 % (35)	0,37 % (1)	1,12 % (3)	1,86 % (5)	7,43 % (20)	6,32 % (17)	1,12 % (3)	100,00 % (269)
A	28	14	115	3	1	4	0	0	0	13	2	2	67,66 % (182)
B	4	3	5	5	0	13	0	0	1	0	2	1	12,64 % (34)
C	3	3	1	0	0	18	1	3	4	7	13	0	19,70 % (53)

Tasks of the type M1.3: determine and represent a numerical expression modeling a situation account for 44.98% (191) of all modeling activities. There is no algebraic potential in 95% (115) of these activities, 4% (5) have a weak potential, and only 1% (1) have a strong potential.

The activities of this type are almost empty of algebraic potential, even if they represent most modeling activities. This result shows that the numerical expressions generated by these activities are exploited only to carry out calculations and not to convey algebraic reasoning.

In the second place, M1.1: Using an algebraic numerical expression to solve a situation model, and M2.1: Using an equation to solve a situation model, each of which accounts for a rate of 13.01%. Almost 90% (31) of the activities of type M2.1 have an algebraic potential, and more than half (18) have a strong algebraic potential, contrary to activities of type M1.1, of which 80% (28) do not represent any algebraic potential, and it is quite normal since these last ones involve numerical expressions. However, type M2.1 activities are distinguished by the presence of an unknown (they are modeled by an equation). The remaining task types are almost non-existent in this textbook. In general, the results from the Al-Moufid textbook show that it contains many modeling activities, but only those derived from situations modeled by a functional relation, or an equation have a significant algebraic potential.

4.3 Characterization of activities according to the criterion of completeness of the modeling process

Among the 269 tasks in which modeling is involved as a capacity to be developed in the student, there are only a few tasks that allow the student to go through the modeling process as described by Chevallard. The following table presents the distribution of the modeling tasks according to the completeness criterion (complete/incomplete modeling).

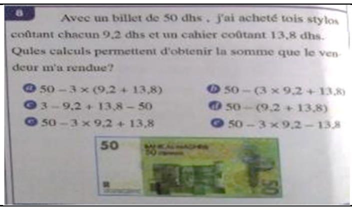
Table 6: distribution of modeling tasks according to the completeness criterion.

	M1	M2	M3	Total
Complete modeling	13,38% (36)	8,55% (23)	6,4% (17)	28,25 % (76)
Incomplete modeling	55,4% (149)	6% (16)	10,4% (28)	71,75 % (34)
Total	68,8% (185)	14,5% (39)	16,7% (45)	100 % (269)

The results presented in table 3, related to the completeness of the modeling process, indicate that among the 185 activities belonging to type M1praxeology: modeling of situations by numerical or algebraic expressions, only 36 allow the student to go through the complete modeling process. This may be because most activities are of the connected type and oriented towards the application of calculation rules whose model is given to the student. However, for the disconnected problems, the model is not unique and not easily identifiable by the student. It is only a question of identifying the data of the problem and the operations to be carried out to arrive at the expression. It is the same model situation in the sense of Blum and Leiss, which intervenes in all the problems of this type.

On the other hand, most activities, which fall under the two praxeologies of the two types M2: modeling by an equation and M3: modeling by a functional relation, allow students to go through the three stages of modeling described by Chevallard (23 out of 39 for M2; 17 out of 28 for M3). These results show that the completeness of the modeling influences positively the algebraic potential since the types of tasks that fall under the praxeology of M2 and M3 represent more algebraic potential compared to M1 tasks. We present some illustrations of the modeling activities that appear in this manual:

1- M1 : modeling of situations by numerical or algebraic expressions

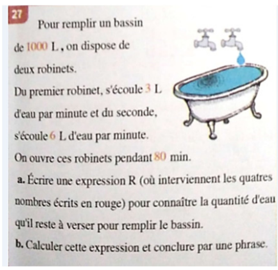

8 Avec un billet de 50 dhs , j'ai acheté tois stylos coûtant chacun 9,2 dhs et un cahier coûtant 13,8 dhs. Qules calculs permettent d'obtenir la somme que le vendeur m'a rendue?

a) 50 – 3 × (9,2 + 13,8)
b) 50 – (3 × 9,2 + 13,8)
c) 3 – 9,2 + 13,8 – 50
d) 50 – (9,2 + 13,8)
e) 50 – 3 × 9,2 + 13,8
f) 50 – 3 × 9,2 – 13,8

Translation:
With a 50 Dirham bill, I bought three pens costing 9.2 Dirhams each and a notebook costing 13,8 Dirhams. What are the calculations to obtain the sum that the vendor gave me back?

Figure 3: extract from the official student manual "Al-Moufid"

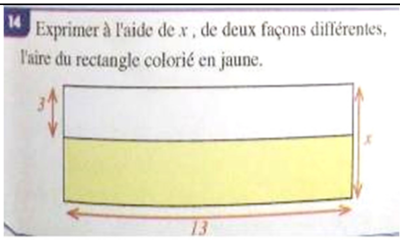
This activity is part of the application exercises, which aim to apply calculation rules. The situation is extra-mathematical, the problem is connected, and the task is of type M1.2: Recognize a numerical expression modeling a given situation. The modeling is not complete since the model is given to the student. The algebraic potential is weak since the instruction does not encourage any algebraic method. However, the numerical expressions proposed allow the student to process their meaning by connecting them to the context of the situation. It also allows the student to identify the equivalent numerical expressions modeling the problem: 50 - (3 x 9.2 +13.8) and 50 - 3 x 9.2 - 13.8


27 Pour remplir un bassin de 1000 L , on dispose de deux robinets.
Du premier robinet, s'écoule 3 L d'eau par minute et du seconde, s'écoule 6 L d'eau par minute.
On ouvre ces robinets pendant 80 min.
a. Écrire une expression R (où interviennent les quatres nombres écrits en rouge) pour connaître la quantité d'eau qu'il reste à verser pour remplir le bassin.
b. Calculer cette expression et conclure par une phrase.

Translation:
Two taps are available to fill a 1000 L pool.
From the first tap 3 L of water flows in one minute, and from the second tap 6 L of water flows per minute.
We open these taps for 80 minutes.
a. Write an expression R (where the four written in red intervene) to know the quantity of water which remains to be poured to fill the basin.
b. Calculate this expression and conclude with a sentence

Figure 4: extract from the official student manual "Al-Moufid"

This is a problem that appears in the problem-solving section. It is an extra-mathematical situation modeled by a numerical expression. The model is given to the student since the first question guides him to use the data from the situation to find the quantity of water necessary to fill the basin. The problem is connected, and the task is of type M1.1: Solve a situation modeled by a numerical expression. The modeling is incomplete since the model is given to the student, and the instruction does not encourage any algebraic method. This problem has weak algebraic potential since it encourages the understanding of the notion of equivalence through the diversity of possible expressions. The 2nd question invites the students to draw a conclusion from the results found. It allows the student to communicate the solution in a real-life situation.


14 Exprimer à l'aide de x , de deux façons différentes, l'aire du rectangle colorié en jaune.

Translation:
Express with x, in two different methods, the area of the rectangle colored in yellow

Figure 5: extract from the official student manual "Al-Moufid"

The problem is part of the part devoted to the deepening of the acquired knowledge, it is an intramathematical situation resulting from geometry which is used to prove the distributivity property $13(x-3) = 13 \times x - 13 \times 3$, the task is of type M1. 1 : Solve a situation modeled by a numerical or algebraic expression, the main task invites the students to find an algebraic equality that allows to deal with the distributivity property, the register is algebraic because the width of the rectangle is designated by the letter x, the modeling is not complete since the model is given to the student and the algebraic potential is strong because on the one hand the register is algebraic since the instruction allows to operate on an indeterminate without taking into account its value, and on the other hand, the situation favors the comprehension of the notion of equivalence between two algebraic expressions and allows to deduce an algebraic property (distributivity of multiplication).

2- M2: modeling of situations by an equation

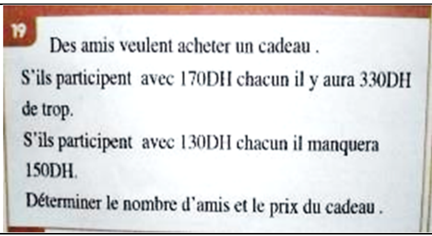
 19 Des amis veulent acheter un cadeau . S'ils participent avec 170DH chacun il y aura 330DH de trop. S'ils participent avec 130DH chacun il manquera 150DH. Déterminer le nombre d'amis et le prix du cadeau .	Translation: Friends want to buy a gift. If they participate with 170 Dirhams each there will be 330 Dirhams too much If they participate with 150 Dirhams each there will be 150 Dirhams missing. Determine the number of friends and the amount of the gift

Figure 6: extract from the official student manual "Al-Moufid"

The problem is part of the problems section. It is a non-mathematical situation based on reality. The task leads the student to determine two unknowns: the number of friends n and the price of the gift p. But the value of the first unknown allows us to determine the second one. The two sentences of the problem lead to the equation 170n-330=130n+150, which will allow us to determine the number of friends (n=12). The gift price can then be calculated using a simple substitution: 170 x 12 - 330 = 1710 or 130 x12 - 150 = 1710. 1: Solve a situation modeled by an equation. The main task does not direct the students towards any arithmetic technique and favors algebraic reasoning. The modeling is complete since the model is not explained, and the algebraic potential is strong since the instruction allows operating on an indeterminate without considering its value.

M3: modeling situations by a functional relationship

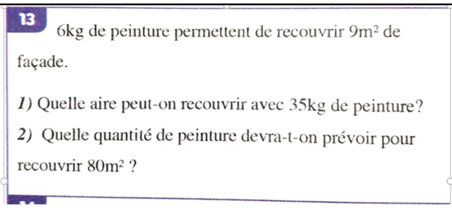
 13 6kg de peinture permettent de recouvrir 9m² de façade. *1)* Quelle aire peut-on recouvrir avec 35kg de peinture? *2)* Quelle quantité de peinture devra-t-on prévoir pour recouvrir 80m² ?	Translation: 6kg of paint can cover 9 m² of facade 1) What area can be covered with 35kg of paint? 2) How much paint will be needed to cover 80 m²?

Figure 7: extract from the official student manual "Al-Moufid"

It is an extra-mathematical situation that presents a functional relation between the painted surface and the quantity of paint used (in Kg). The modeling is complete since the model is not given to the pupil. The instruction invites the pupil to go through the process of modeling by identifying the mathematical model, first through the identification of the relevant data of the situation (6 Kg and 9m^2), and then to deduce the proportional relation between the two variables: area = 1.5 x mass. This relationship can be represented in several semantic registers, in the form of a proportionality table or a graph...

5. Conclusion and Discussion

This article aims to determine the institutional reports on modeling as the main component of algebraic thinking at the beginning of middle school. The mathematics curriculum at this school level is represented by the only document entitled "pedagogical guidelines for teaching mathematics in middle school." Reading this document shows that the word "modeling" is mentioned only once as a capacity to be developed in the middle school student, and no expression mentions a definition of modeling as a process that develops algebraic thinking.

The results of analyzing the Al-Moufid textbook indicate that it contains a significant number of modeling activities, most of them of the type M1: Modeling a situation through a numerical or algebraic expression. These tasks were oriented by the designers of the textbook towards the execution of calculations and not towards the development of algebraic reasoning. According to Najar et al. (2021), this can be explained by the ubiquity of connected problems compared to disconnected problems, which represent a very high degree of analycity. This result represents a key vector that can assist designers in directing this textbook toward developing algebraic thinking by offering disconnected problems that encourage students to operate on an unknown rather than the naming quality of the data, and where computation is viewed as a tool rather than an end.

On the other hand, tasks of type M2 and M3 are very few, and yet they represent important algebraic potential compared to tasks of type M1. This can be explained by the presence of an indeterminate that may or may not be represented by a symbol (unknown; variable) on which the student must operate. In this respect, it is essential to multiply this kind of task, especially before the introduction of the letter, to prepare them to feel the importance of using a symbol to designate the indeterminate**.**

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